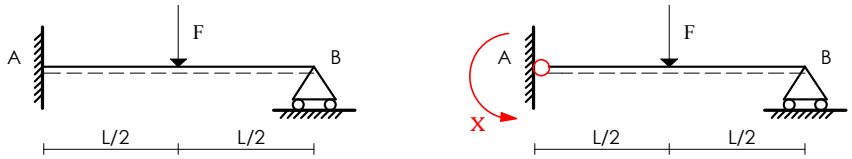
Tipologie di strutture iperstatiche dell'eserciziario:

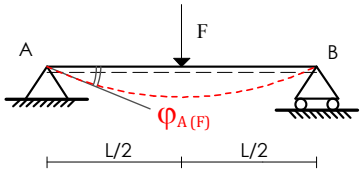
Tutte le strutture affrontate nel presente eserciziario possono essere sostanzialmente ricondotte a 3 tipologie principali. Di seguito si riportano i più opportuni svincolamenti e le relative Eq. di Congruenza tipo.

1° Tipologia: Trave con incastro e carrello. Lo svincolamento piu' opportuno prevede la trasformazione dell'incastro in cerniera e la congruenza come sommatoria di rotazioni nulle in corrispondenza del punto di svincolamento.



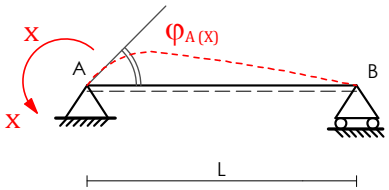
Eq. di Congruenza: $\boxed{\varphi_A = 0} \Rightarrow \varphi_{A\,(F)} + \varphi_{A\,(X)} = 0$

$\boxed{\varphi_{A\,(F)}} \Rightarrow$



$$\boxed{\varphi_{A\,(FL)} = -\frac{FL^2}{16\,EJ}}$$

$\boxed{\varphi_{A\,(X)}} \Rightarrow$



$$\boxed{\varphi_{A\,(X)} = \frac{XL}{3\,EJ}}$$

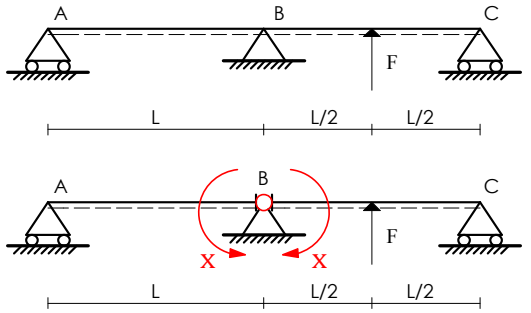
Eq. di Congruenza:

$$\varphi_{A\,(F)} + \varphi_{A\,(X)} = 0 \quad \Rightarrow \quad -\frac{FL^2}{16\,EJ} + \frac{XL}{3\,EJ} = 0$$

$$X = \frac{FL^2}{16\,EJ} \cdot \frac{3\,EJ}{L} = \boxed{3/16FL}$$

In questa tipolgia di strutture l'incognita iperstatica X rappresenta il valore della reazione vincolare M_A dell'incastro.

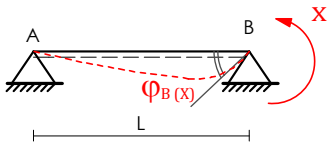
2° Tipologia: Trave su tre appoggi. Lo svincolamento piu' opportuno prevede l'introduzione di una sconnessione interna (cerniera)in corrispondenza dell'appoggio centrale. In questi casi la struttura principale e' costituita dai due tronchi in cui la cerniera interna suddivide la trave originaria



La congruenza sara' data dall'imposizione che la rotazione relativa tra i due tronchi sia nulla. $\Delta\varphi = 0$

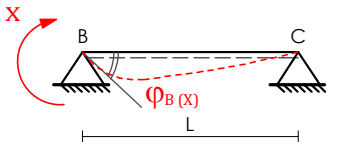
Eq. di Congruenza: $\boxed{\Delta\varphi_B = 0} \Rightarrow \boxed{\varphi_{BA} = \varphi_{BC}} \Rightarrow \varphi_{BA\,(X)} = \varphi_{BC\,(X)} + \varphi_{BC\,(3F)}$

$\boxed{\varphi_{BA\,(X)}} \Rightarrow$



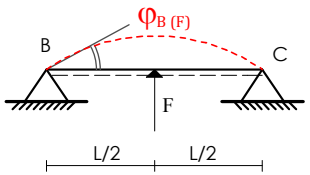
$$\boxed{\varphi_{BA\,(X)} = \frac{XL}{3\,EJ}}$$

$\boxed{\varphi_{BC\,(X)}} \Rightarrow$



$$\boxed{\varphi_{BC\,(X)} = -\frac{XL}{3\,EJ}}$$

$\boxed{\varphi_{BC\,(X)}} \Rightarrow$



$$\boxed{\varphi_{BC\,(F)} = \frac{FL^2}{16\,EJ}}$$

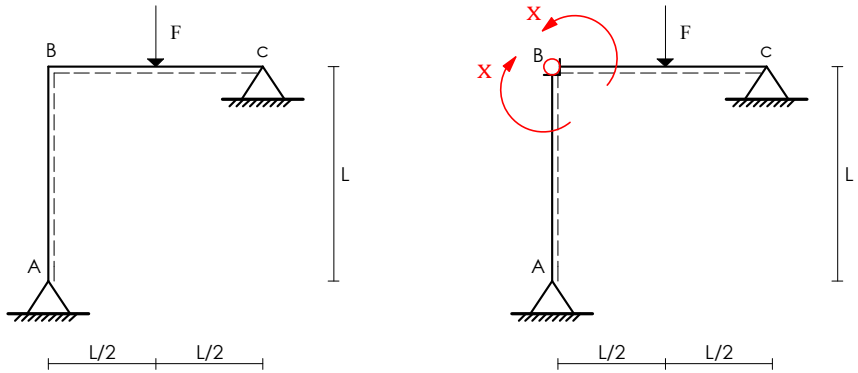
Eq. di Congruenza:

$$\varphi_{BA\,(X)} = \varphi_{BC\,(X)} + \varphi_{BC\,(F)} \quad \Rightarrow \quad \frac{XL}{3\,EJ} = -\frac{XL}{3\,EJ} + \frac{FL^2}{16\,EJ}$$

$$X = \frac{FL^2}{16\,EJ} \cdot \frac{3\,EJ}{2L} = \boxed{3/32\,FL}$$

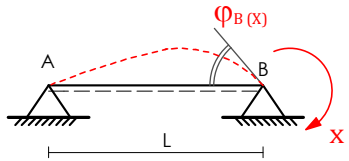
In questa tipolgia di strutture l'incognita iperstatica X rappresenta il valore del Momento M in corrispondenza del punto B di svincolamento.

3° Tipologia: Trave con due cerniere esterne. Lo svincolamento piu' opportuno prevede l'introduzione di una sconnessione interna (cerniera)in corrispondenza del punto angoloso. Analogamente al caso precedente, la struttura principale e' costituita dai due tronchi in cui la cerniera interna suddivide la trave originaria.



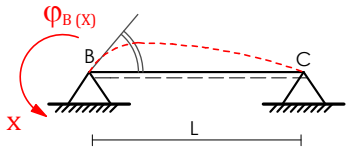
Eq. di Congruenza: $\Delta\varphi_B = 0 \Rightarrow \varphi_{BA} = \varphi_{BC} \Rightarrow \varphi_{BA\,(X)} = \varphi_{BC\,(X)} + \varphi_{BC\,(F)}$

$\varphi_{BA\,(X)} \Rightarrow$



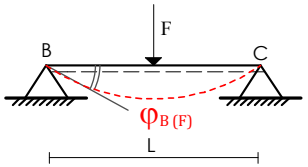
$\varphi_{BA\,(X)} = -\dfrac{XL}{3\,EJ}$

$\varphi_{BC\,(X)} \Rightarrow$



$\varphi_{BC\,(X)} = \dfrac{XL}{3\,EJ}$

$\varphi_{BC\,(X)} \Rightarrow$



$\varphi_{BC\,(F)} = -\dfrac{FL^2}{16\,EJ}$

Eq. di Congruenza:

$$\varphi_{BA\,(X)} = \varphi_{BC\,(X)} + \varphi_{BC\,(F)} \Rightarrow -\frac{XL}{3\,EJ} = \frac{XL}{3\,EJ} - \frac{FL^2}{16\,EJ}$$

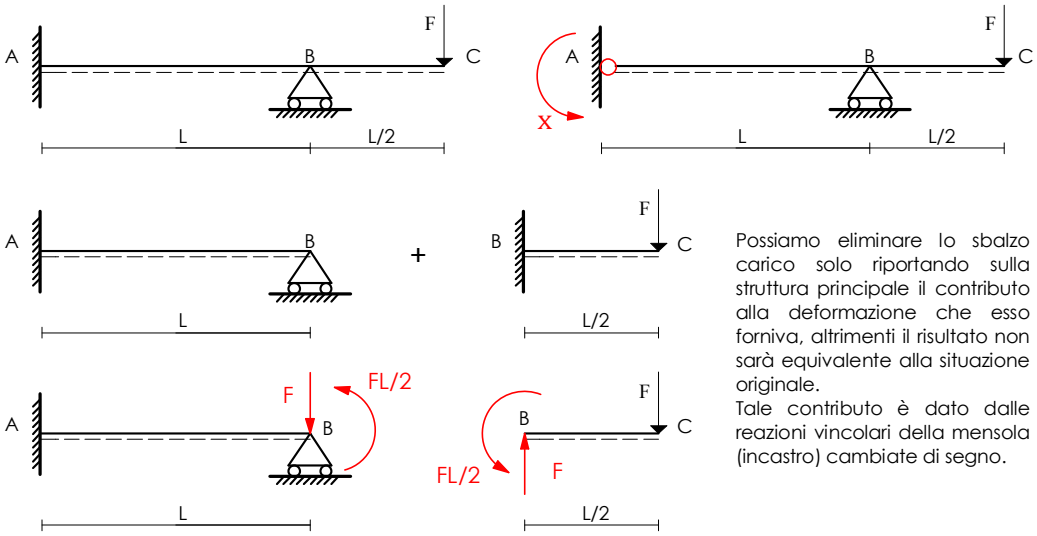
$$X = \frac{FL^2}{16\,EJ} \cdot \frac{3\,EJ}{2L} = \boxed{3/32\,FL}$$

Anche in questi casi, l'incognita iperstatica X rappresenta il valore del Momento M in corrispondenza del punto B di svincolamento.

Come eliminare le appendici isostatiche:

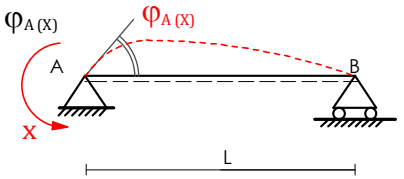
Prima di iniziare con lo svolgimento degli esercizi e' importante fare alcune considerazioni relative al metodo per eliminare le eventuali appendici isostatiche di strutture iperstatiche.
Tale operazione è necessaria per individuare la struttura principale su cui attuare lo svincolamento e la successiva congruenza.
Affinchè la situazione che si viene a creare, a seguito dell'eliminazione di una appendice isostatica, sia equivalente a quella di partenza è necessario tenere conto del contributo che tale appendice fornisce alla deformazione della struttura e quindi alle rotazioni/abbassamenti che si andranno a calcolare con le equazioni di congruenza.
Le appendici isostatiche possono essere equiparate a delle mensole e il punto di innesto (corrispondente ad un tratto continuo) vincolato o meno, può essere equiparato ad un incastro, di cui ci sono note tutte le reazioni vincolari.

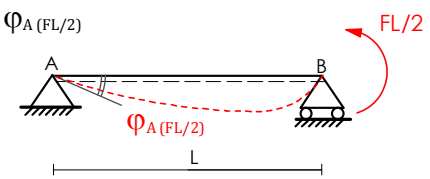
Consideriamo ad esempio la seguente struttura iperstatica costituita da una trave vincolata nel punto A da un incastro e nel punto B da un carrello e dotata di una appendice isastatica (sbalzo BC) caricato all'estremità con una forza concentrata F. La forza F determina sulla porzione AB una deformazione.
La porzione di struttura AB è la struttura principale su cui operare lo svincolamento, ad es. trasformando l'incastro in una cerniera. Per ricorrere ai risultati notevoli, dobbiamo ricondurre la struttura a quella principale eliminando lo sbalzo carico e ottenendo così una trave su due appoggi (AB).

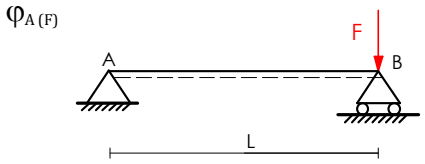


Possiamo eliminare lo sbalzo carico solo riportando sulla struttura principale il contributo alla deformazione che esso forniva, altrimenti il risultato non sarà equivalente alla situazione originale.
Tale contributo è dato dalle reazioni vincolari della mensola (incastro) cambiate di segno.

Analizziamo ai fini della congruenza i singoli contributi alla deformazione della struttura principale AB (nel caso specifico la rotazione in A, φ_A) di ciascun carico.





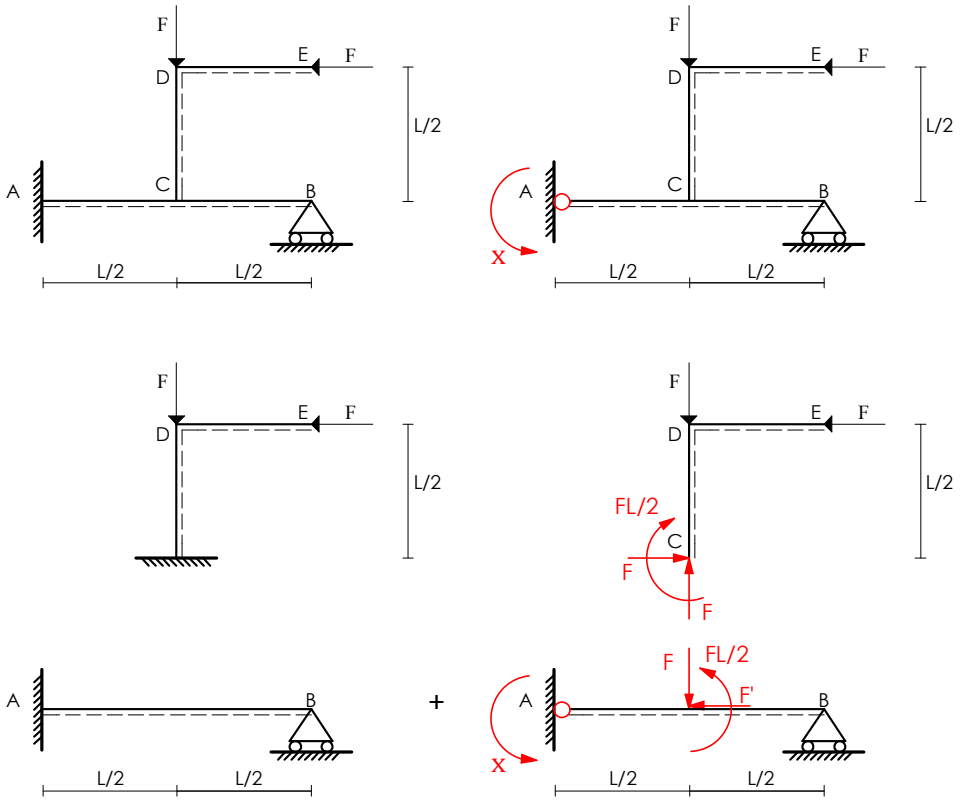


Osservando la figura a lato si nota che la forza F non contribuisce alla deformazione della struttura AB in quanto assorbita dal vincolo in B (carrello) mentre lo sforzo di momento FL/2 , generato dalla forza F per il braccio L/2 contribuiva alla deformazione. La forza concentrata F può dunque non essere considerata ai fini della congruenza.

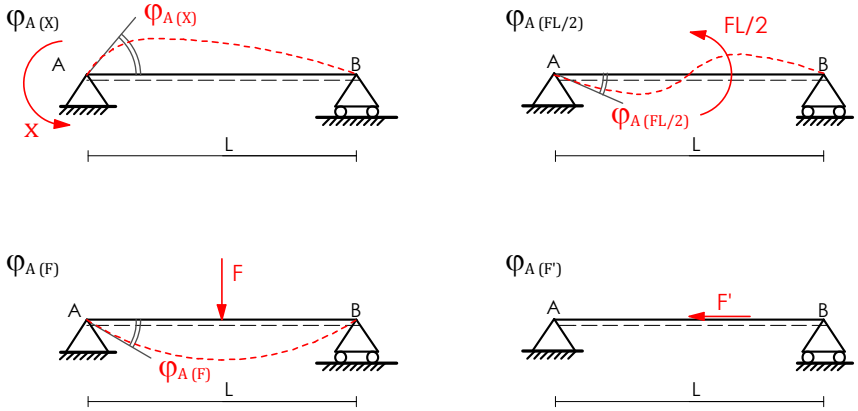
Eq. di Congruenza: $\boxed{\varphi_A = 0}$ $\Rightarrow$ $\varphi_{A\,(FL/2)} + \varphi_{A\,(X)} = 0$

Non sempre la forza concentrata di uno sbalzo carico non contribuisce alla deformazione della struttura principale. Questo avviene infatti solo nei casi in cui lo sbalzo carico viene eliminato in corrispondenza di un vincolo. A maggior chiarimento vediamo un caso in cui è necessario considerare anche il contributo della forza concentrata oltre al contributo di momento che essa genera rispetto al punto in cui lo sbalzo viene eliminato.

Consideriamo una struttura analoga alla precedente, ma nella quale l'appendice isostatica da eliminare CDE si innesta nella campata libera della trave e non in corrispondenza di un vincolo.
La struttura principale è sempre AB.



Analizziamo ora ai fini della congruenza i singoli contributi alla deformazione della struttura principale AB (nel caso specifico la rotazione in A, φ_A) di ciascun carico.

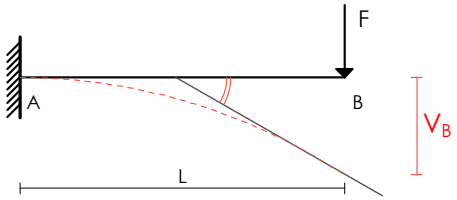


Eq. di Congruenza: $\boxed{\varphi_A = 0}$ $\Rightarrow$ $\varphi_{A\,(X)} + \varphi_{A\,(FL/2)} + \varphi_{A\,(F)} = 0$

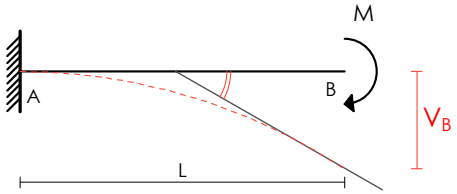
Appare evidente che, in questo caso, la forza concentrata F determini una deformazione della struttura principale e dia quindi un contributo alla rotazione φ_A . Solo la forza F' non contribuisce alla rotazione in A e non verrà considerata nell'eq. di congruenza.

Per evitare di non considerare tutti i contributi necessari è sempre meglio riportare l'intero sistema delle reazioni vincolari della appendice isostatica eliminata, cambiate di segno, e valutare a posteriori quali reazioni contribuiscano o meno alla deformazione della struttura principale.

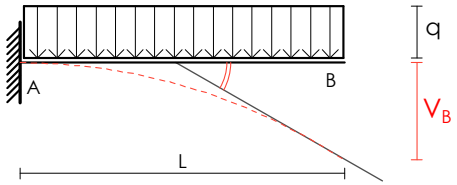
TABELLA DEGLI ABBASSAMENTI E ROTAZIONI NOTEVOLI



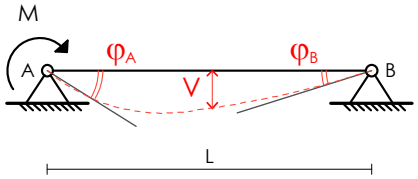
$\varphi_B = \frac{FL^2}{2EJ}$ $\quad V_B = \frac{FL^3}{3EJ}$



$\varphi_B = \frac{M(L)}{EJ}$ $\quad V_B = \frac{F(L^2)}{2\,EJ}$

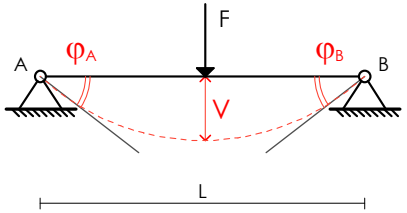


$\varphi_B = \frac{q(L^3)}{6\,EJ}$ $\quad V_B = \frac{q(L^4)}{8\,EJ}$

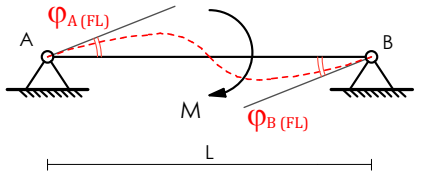


$\varphi_A = \frac{M(L)}{3\,EJ}$ $\quad V = \frac{M(L^2)}{16\,EJ}$

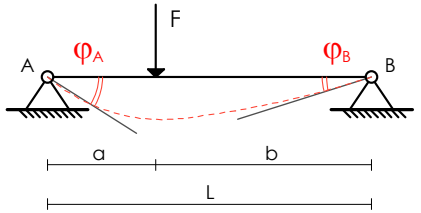
$\varphi_B = \frac{M(L)}{6\,EJ}$



$\varphi_A = \varphi_B = \frac{F(L^2)}{16\,EJ}$ $\quad V = \frac{F(L^3)}{48\,EJ}$

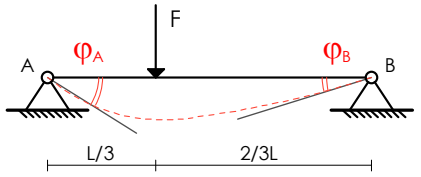


$\varphi_A = \varphi_B = \frac{M(L)}{24\,EJ}$



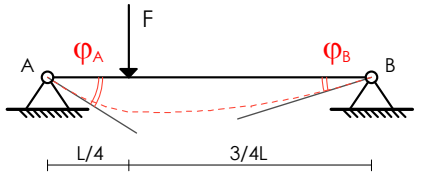
$\varphi_A = \frac{Fb\,(L^2 - b^2)}{3\,EJ(L)}$

$\varphi_B = \frac{Fa\,(L^2 - a^2)}{3\,EJ(L)}$



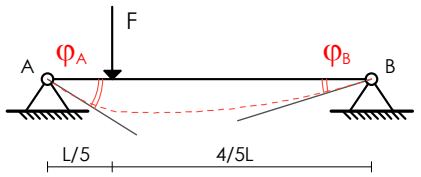
$\varphi_A = \frac{5F(L^2)}{81\,EJ}$

$\varphi_B = \frac{4F(L^2)}{81\,EJ}$



$\varphi_A = \frac{7F(L^2)}{128\,EJ}$

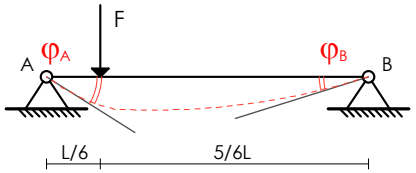
$\varphi_B = \frac{5F(L^2)}{128\,EJ}$



$\varphi_A = \frac{6F(L^2)}{125\,EJ}$

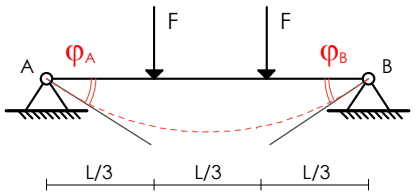
$\varphi_B = \frac{4F(L^2)}{125\,EJ}$

TABELLA DEGLI ABBASSAMENTI E ROTAZIONI NOTEVOLI

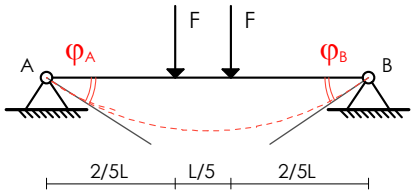


$$\varphi_A = \frac{55F(L^2)}{1296\ EJ}$$

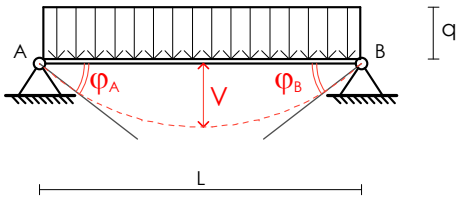
$$\varphi_B = \frac{35F(L^2)}{1296\ EJ}$$



$$\varphi_A = \varphi_B = \frac{FL^2}{9\ EJ}$$

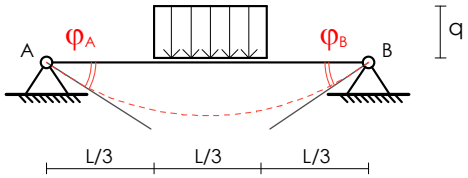


$$\varphi_A = \varphi_B = \frac{3F(L^2)}{25\ EJ}$$

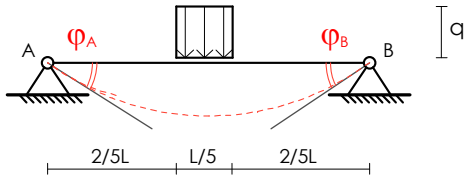


$$\varphi_A = \varphi_B = \frac{q(L^3)}{24\ EJ}$$

$$V = \frac{5q(L^4)}{384\ EJ}$$

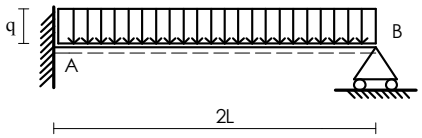


$$\varphi_A = \varphi_B = \frac{13q(L^3)}{648\ EJ}$$



$$\varphi_A = \varphi_B = \frac{37q(L^3)}{3000\ EJ}$$

Esercizio N.1

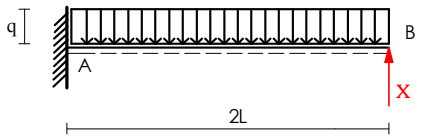


Per risolvere la struttura con il metodo delle forze è possibile procedere in due modi diffferenti:

Soluzione 1: Svincolare in corrispondenza del carrello e calcolare l'abbassamento in B. In questo caso l'incognita iperstatica **X** sarà la reazione vincolare **V** del carrello.

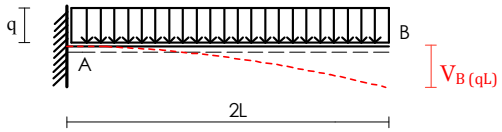
Soluzione 2: Svincolare in corrispondenza dell'incastro, ovvero trasformarlo in una cerniera e calcolare la rotazione in A. In questo caso l'incognita iperstatica **X** sarà invece la reazione vincolare **M** dell'incastro.

SOLUZIONE 1: abbassamenti in B



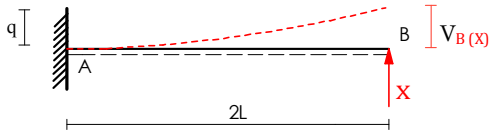
Eq. di Congruenza: $\boxed{V_B = 0} \Rightarrow V_{B\,(qL)} + V_{B\,(X)} = 0$

$\boxed{V_{B\,(qL)}} \Rightarrow$



$$\boxed{V_{B\,(qL)} = \frac{q(2L)^4}{8\,EJ}}$$

$\boxed{V_{B\,(X)}} \Rightarrow$



$$\boxed{V_{B\,(X)} = -\frac{X(2L)^3}{3\,EJ}}$$

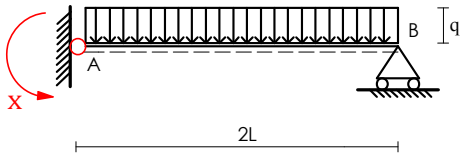
Eq. di Congruenza:

$$V_{B\,(qL)} + V_{B\,(X)} = 0 \quad \Rightarrow \quad \frac{q(2L)^4}{8\,EJ} - \frac{X(2L)^3}{3\,EJ} = 0$$

$$X = \frac{q(2L)^4}{8\,EJ} \cdot \frac{3\,EJ}{(2L)^3} = \boxed{3/4\,qL}$$

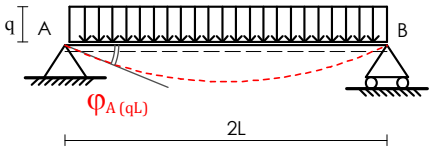
Esercizio N.1

SOLUZIONE 2: rotazioni in A



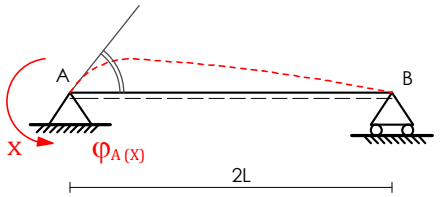
Eq. di Congruenza: $\boxed{\varphi_A = 0} \Rightarrow \varphi_{A\,(qL)} + \varphi_{A\,(X)} = 0$

$\boxed{\varphi_{A\,(qL)}} \Rightarrow$



$$\boxed{\varphi_{A\,(qL)} = -\frac{q(2L)^3}{24\,EJ}}$$

$\boxed{\varphi_{A\,(X)}} \Rightarrow$



$$\boxed{\varphi_{A\,(X)} = \frac{2XL}{3\,EJ}}$$

Eq. di Congruenza:

$$\varphi_{A\,(qL)} + \varphi_{A\,(X)} = 0 \quad \Rightarrow \quad -\frac{qL^3}{3\,EJ} + \frac{2XL}{3\,EJ} = 0$$

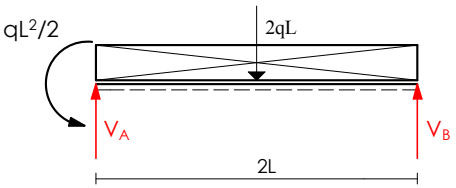
$$X = \frac{qL^3}{3\,EJ} \cdot \frac{3\,EJ}{2L} = \boxed{qL^2/2}$$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B

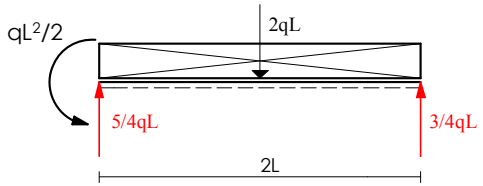
$\Sigma M_B = 0$
$-\,qL^2/2 + V_B \cdot 2L = 0 \rightarrow \boxed{V_B = 3/4qL}$

$\Sigma V = 0$
$V_A + V_B - 2qL = 0 \rightarrow \boxed{V_A = 5/4qL}$

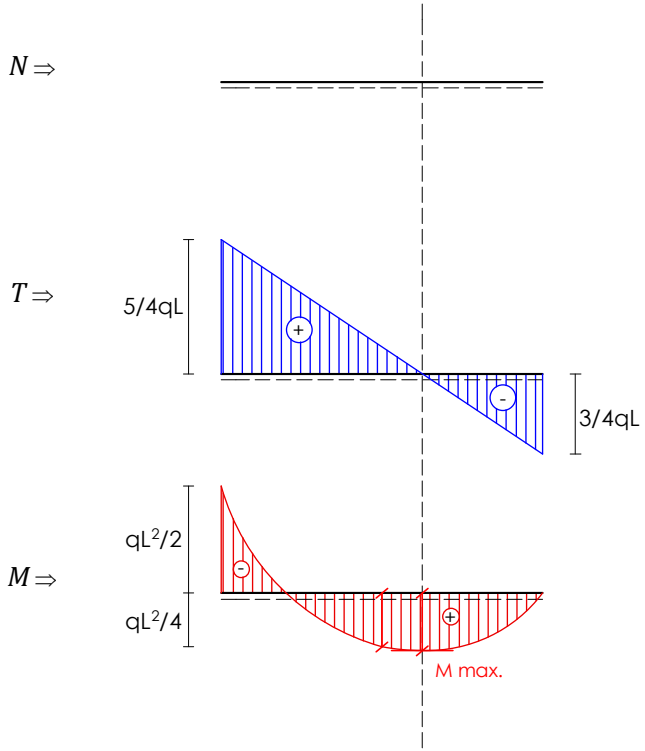


Esercizio N.1

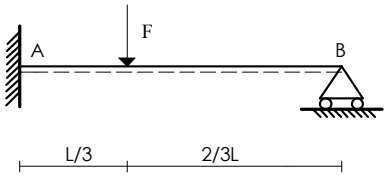
Diagramma di corpo libero :



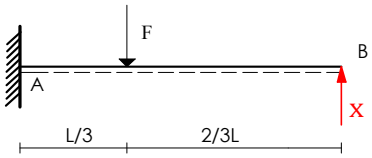
Diagrammi (N, T, M) :



Esercizio N.2

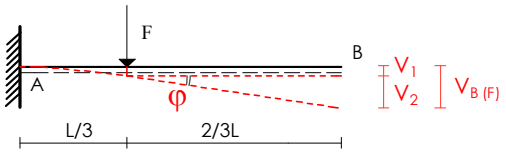


SOLUZIONE 1: abbassamenti in B



Eq. di Congruenza: $\boxed{V_B = 0} \Rightarrow V_{B\,(F)} + V_{B\,(X)} = 0$

$\boxed{V_{B\,(F)}} \Rightarrow$



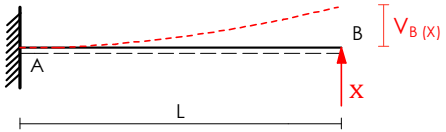
$\boxed{V_{B\,(F)} = V_1 + V_2 \rightarrow}$

$$\frac{F(L/3)^3}{3\,EJ} + \varphi \cdot 2/3L =$$

$$\frac{F(L/3)^3}{3\,EJ} + \frac{F(L/3)^2}{2\,EJ} \cdot 2/3L =$$

$$\boxed{V_{B(F)} = \frac{4FL^3}{81\,EJ}}$$

$\boxed{V_{B\,(X)}} \Rightarrow$



$$\boxed{V_{B\,(X)} = -\frac{XL^3}{3\,EJ}}$$

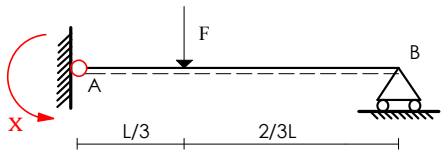
Eq. di Congruenza:

$$V_{B\,(F)} + V_{B\,(X)} = 0 \quad \Rightarrow \quad \frac{4FL^3}{81\,EJ} - \frac{XL^3}{3\,EJ} = 0$$

$$X = \frac{4FL^3}{81\,EJ} \cdot \frac{3\,EJ}{L^3} = \boxed{4/27\ qL}$$

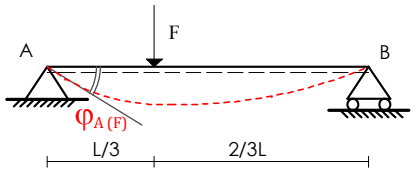
Esercizio N.2

SOLUZIONE 2: rotazioni in A



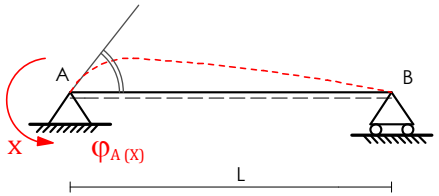
Eq. di Congruenza: $\boxed{\varphi_A = 0} \Rightarrow \varphi_{A(F)} + \varphi_{A(X)} = 0$

$\boxed{\varphi_{A(F)}} \Rightarrow$



$$\boxed{\varphi_{A(F)} = -\frac{5FL^2}{81\,EJ}}$$

$\boxed{\varphi_{A(X)}} \Rightarrow$



$$\boxed{\varphi_{A(X)} = \frac{XL}{3\,EJ}}$$

Eq. di Congruenza:

$$\varphi_{A(F)} + \varphi_{A(X)} = 0 \quad \Rightarrow \quad -\frac{5FL^2}{81\,EJ} + \frac{XL}{3\,EJ} = 0$$

$$X = \frac{5FL^2}{81\,EJ} \cdot \frac{3\,EJ}{L} = \boxed{5/27FL}$$

Calcolo reazioni vincolari incognite:

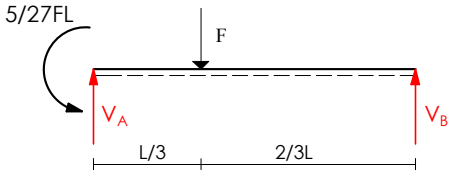
Eq. Ausiliaria : M_B

$\Sigma M_B = 0$

$5/27FL + V_B \cdot L - F \cdot L/3 = 0 \rightarrow \boxed{V_B = 4/27F}$

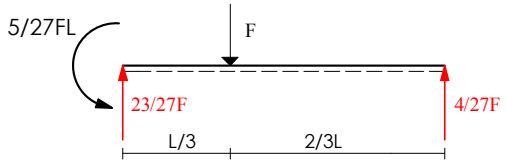
$\Sigma V = 0$

$V_A + V_B - F = 0 \rightarrow \boxed{V_A = 23/27F}$



Esercizio N.2

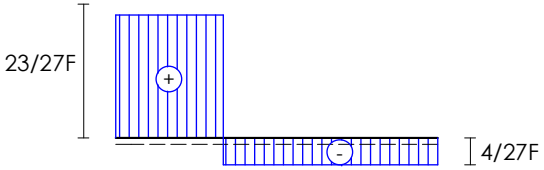
Diagramma di corpo libero :



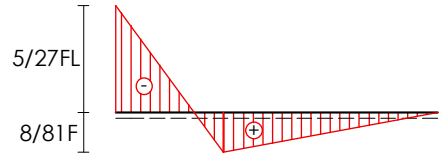
Diagrammi (N, T, M) :

$N \Rightarrow$

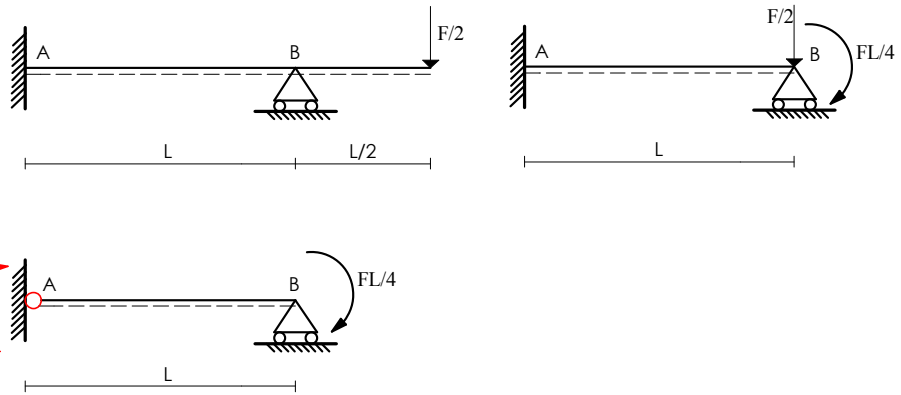
$T \Rightarrow$



$M \Rightarrow$

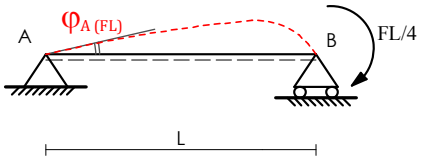


Esercizio N.3



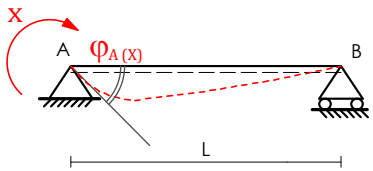
Eq. di Congruenza: $\boxed{\varphi_A = 0} \Rightarrow \varphi_{A\,(FL)} + \varphi_{A\,(X)} = 0$

$\boxed{\varphi_{A\,(FL)}} \Rightarrow$



$$\boxed{\varphi_{A\,(FL)} = -\frac{FL^2/4}{6\,EJ}}$$

$\boxed{\varphi_{A\,(X)}} \Rightarrow$



$$\boxed{\varphi_{A\,(X)} = \frac{XL}{3\,EJ}}$$

Eq. di Congruenza:

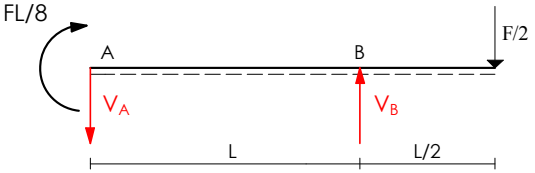
$$\varphi_{A\,(FL)} + \varphi_{A\,(X)} = 0 \Rightarrow \frac{FL^2/4}{6\,EJ} - \frac{XL}{3\,EJ} = 0$$

$$X = \frac{FL^2}{24\,EJ} \cdot \frac{3\,EJ}{L} = \boxed{FL/8}$$

Esercizio N.3

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B



$\Sigma M_B = 0$

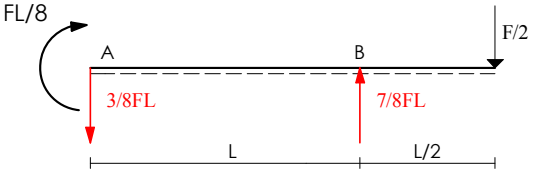
$- FL/8 + V_B \cdot L - F/2 \cdot 3/2L = 0$

$\rightarrow \boxed{V_B = 7/8FL}$

$\Sigma V = 0$

$V_A - V_B + F/2 = 0 \rightarrow \boxed{V_A = 3/8F}$

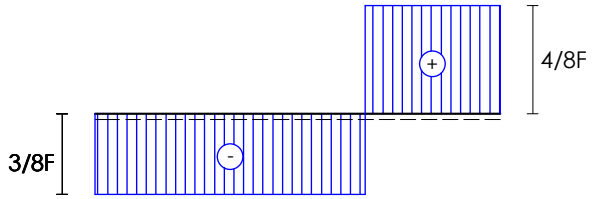
Diagramma di corpo libero :



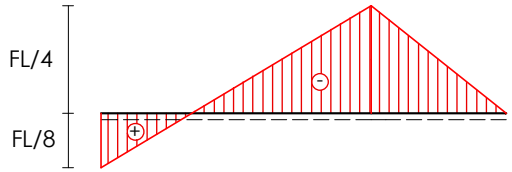
Diagrammi (N, T, M) :

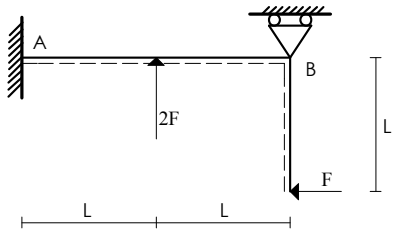
$N \Rightarrow$

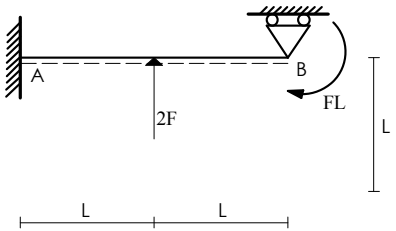
$T \Rightarrow$

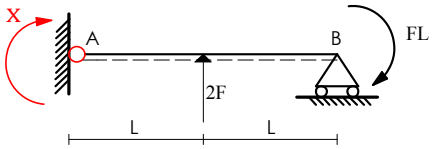


$M \Rightarrow$



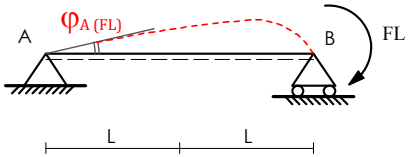






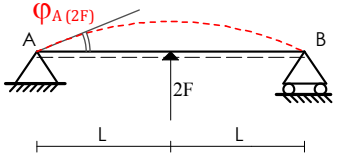
Eq. di Congruenza: $\boxed{\varphi_A = 0} \Rightarrow \varphi_{A\,(FL)} + \varphi_{A\,(2F)} + \varphi_{A\,(X)} = 0$

$\boxed{\varphi_{A\,(FL)}} \Rightarrow$



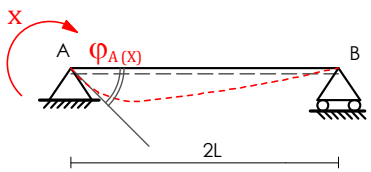
$$\varphi_{A\,(FL)} = \frac{F(2L)^2}{6\,EJ}$$

$\boxed{\varphi_{A\,(2F)}} \Rightarrow$



$$\varphi_{A\,(X)} = \frac{2F(2L)^2}{16\,EJ}$$

$\boxed{\varphi_{A\,(X)}} \Rightarrow$



$$\varphi_{A\,(X)} = -\frac{X2L}{3\,EJ}$$

Esercizio N.4

Eq. di Congruenza:

$$\varphi_{A\,(FL)} + \varphi_{A\,(2F)} + \varphi_{A\,(X)} = 0 \Rightarrow \frac{F(2L)^2}{6\,EJ} + \frac{2F(2L)^2}{16\,EJ} - \frac{X2L}{3\,EJ} = 0$$

$$X = \frac{2FL^2}{3\,EJ} \cdot \frac{3\,EJ}{2L} = \boxed{5/4\,FL}$$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_A

$$\Sigma M_A = 0$$
$$-5/4FL + V_B \cdot 2L + F \cdot L = 0$$
$$\rightarrow \boxed{V_B = F/8}$$

$$\Sigma V = 0$$
$$-V_A + V_B + 2F = 0 \rightarrow \boxed{V_A = 17/8\,F}$$

$$\Sigma H = 0$$
$$H_A - F = 0 \rightarrow \boxed{H_A = F}$$

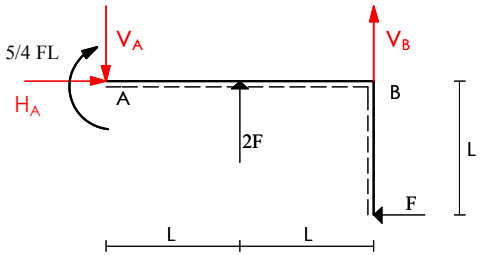
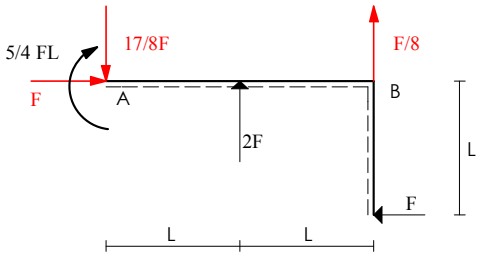
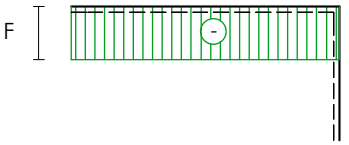


Diagramma di corpo libero :



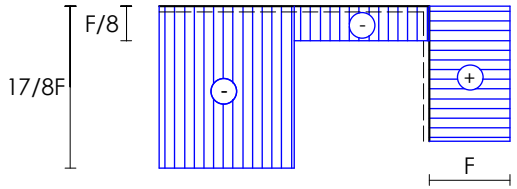
Diagrammi (N, T, M) :

$N \Rightarrow$

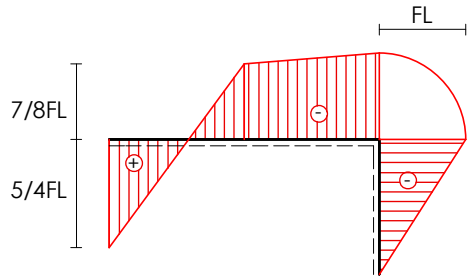


Esercizio N.4

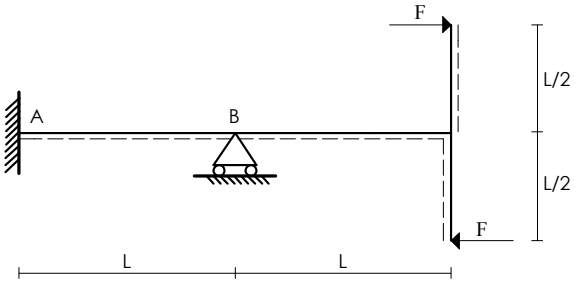
$T \Rightarrow$

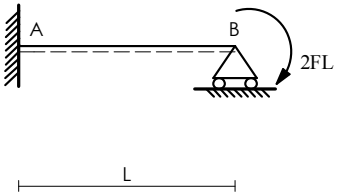


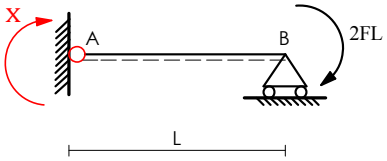
$M \Rightarrow$



Esercizio N.5

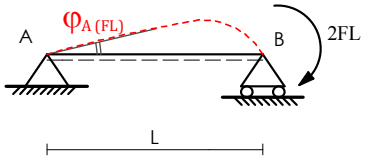






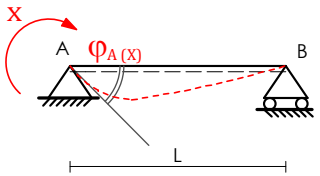
Eq. di Congruenza: $\boxed{\varphi_A = 0} \Rightarrow \varphi_{A\,(FL)} + \varphi_{A\,(X)} = 0$

$\boxed{\varphi_{A\,(FL)}} \Rightarrow$



$$\boxed{\varphi_{A\,(FL)} = \frac{FL^2}{6\,EJ}}$$

$\boxed{\varphi_{A\,(X)}} \Rightarrow$



$$\boxed{\varphi_{A\,(X)} = -\frac{XL}{3\,EJ}}$$

Eq. di Congruenza:

$$\varphi_{A\,(FL)} + \varphi_{A\,(X)} = 0 \quad \Rightarrow \quad \frac{FL^2}{6\,EJ} - \frac{XL}{3\,EJ} = 0$$

$$X = \frac{FL^2}{6\,EJ} \cdot \frac{3\,EJ}{L} = \boxed{FL/2}$$

Esercizio N.5

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_A

$\Sigma M_A = 0$

$- F \cdot L/2 + V_B \cdot L - F \cdot L/2 = 0$

$\rightarrow \boxed{V_B = 3/2F}$

$\Sigma V = 0$

$- V_A + V_B = 0 \rightarrow \boxed{V_A = 3/2\ F}$

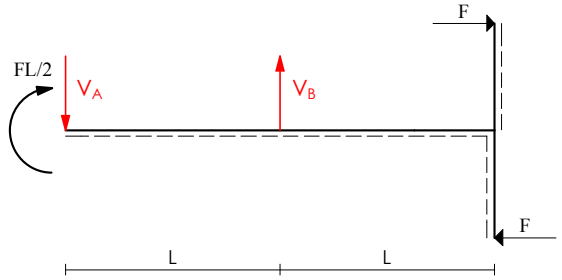
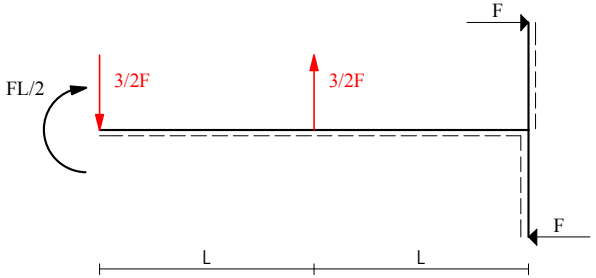
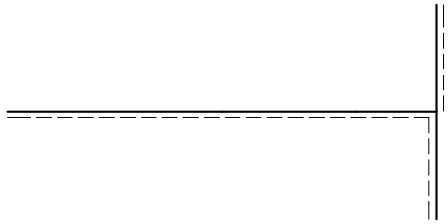


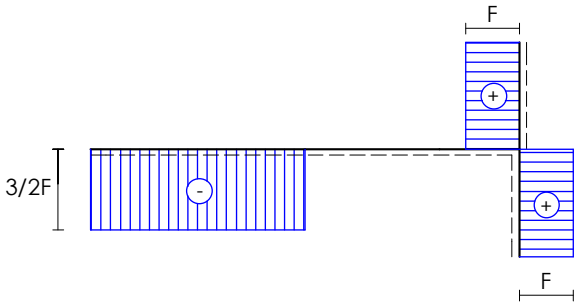
Diagramma di corpo libero :



$N \Rightarrow$

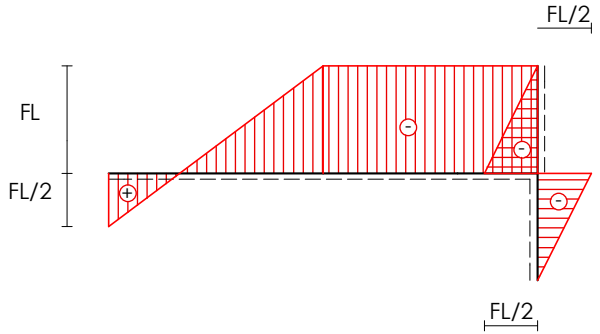


$T \Rightarrow$

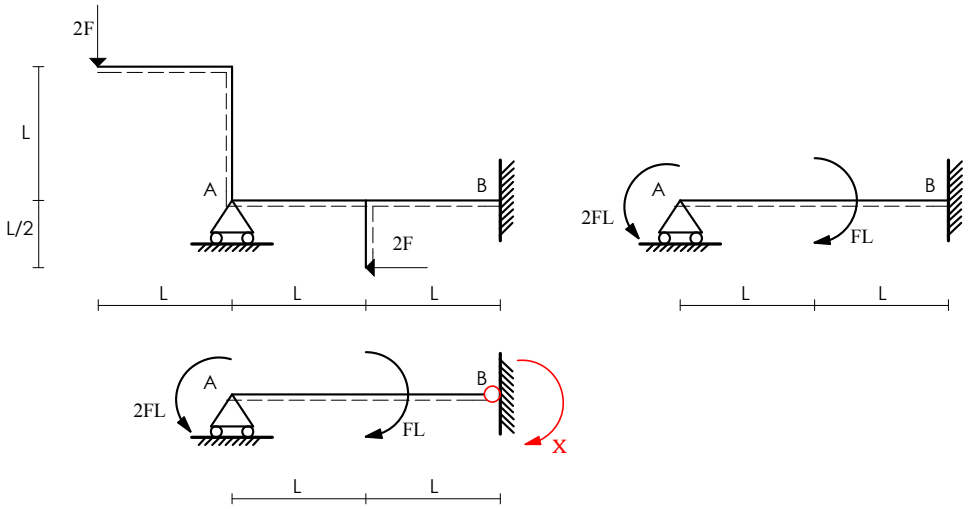


Esercizio N.5

$M \Rightarrow$

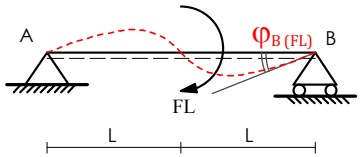


Esercizio N.6



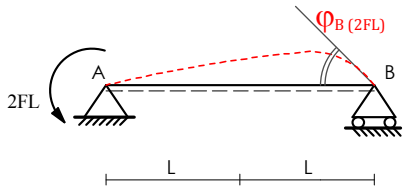
Eq. di Congruenza: $\boxed{\varphi_B = 0} \Rightarrow \varphi_{B\,(FL)} + \varphi_{B\,(2FL)} + \varphi_{B\,(X)} = 0$

$\boxed{\varphi_{B\,(FL)}} \Rightarrow$



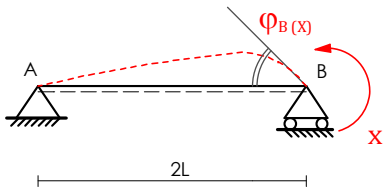
$$\boxed{\varphi_{B\,(FL)} = \frac{FL(2L)}{24EJ}}$$

$\boxed{\varphi_{B\,(2FL)}} \Rightarrow$



$$\boxed{\varphi_{B\,(X)} = -\frac{2FL(2L)}{6EJ}}$$

$\boxed{\varphi_{B\,(X)}} \Rightarrow$



$$\boxed{\varphi_{B\,(X)} = \frac{X2L}{3EJ}}$$

Esercizio N.6

Eq. di Congruenza:

$$\varphi_{B\,(FL)} + \varphi_{B\,(2FL)} + \varphi_{B\,(X)} = 0 \Rightarrow \frac{FL^2}{12\,EJ} - \frac{2FL^2}{3\,EJ} + \frac{X2L}{3\,EJ} = 0$$

$$X = \frac{7FL^2}{12\,EJ} \cdot \frac{3\,EJ}{2L} = \boxed{7/8\,FL}$$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B

$\Sigma M_B = 0$
$7/8FL - V_A \cdot 2L + 2F \cdot 3L - 2F \cdot L/2 = 0$
$\rightarrow \boxed{V_A = 47/16FL}$

$\Sigma V = 0$
$V_A - V_B - 2F = 0 \rightarrow \boxed{V_B = 15/16F}$

$\Sigma H = 0$
$H_A - 2F = 0 \rightarrow \boxed{H_A = 2F}$

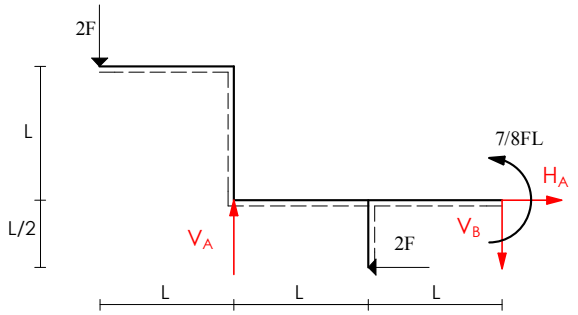
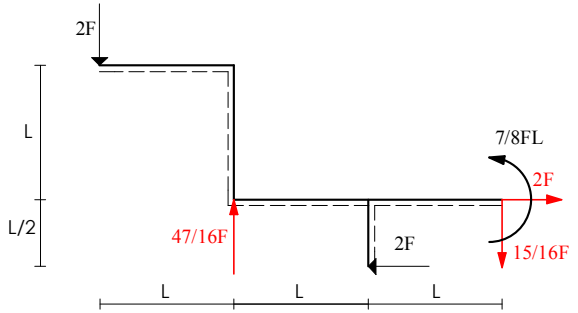
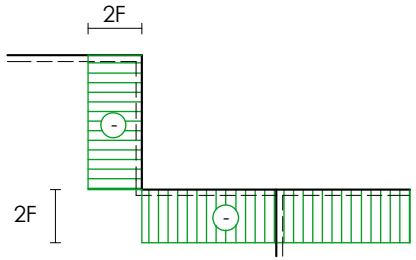


Diagramma di corpo libero :

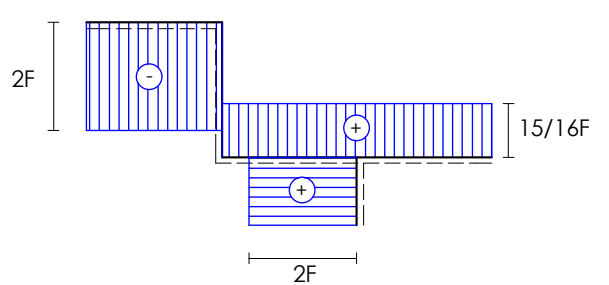


Diagrammi (N, T, M) :

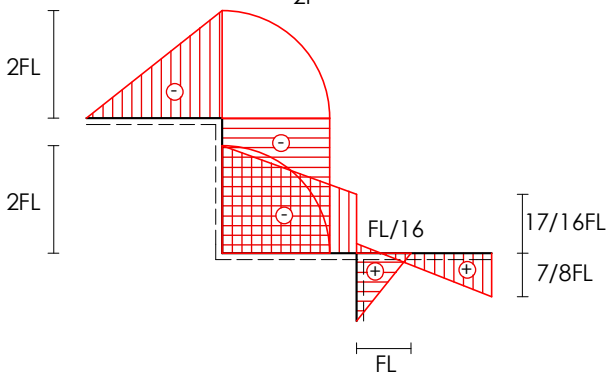
$N \Rightarrow$



$T \Rightarrow$

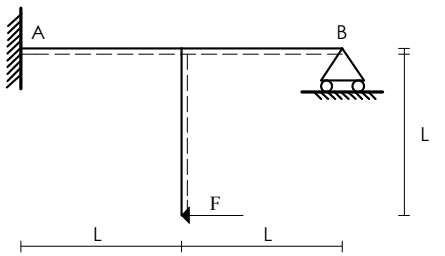


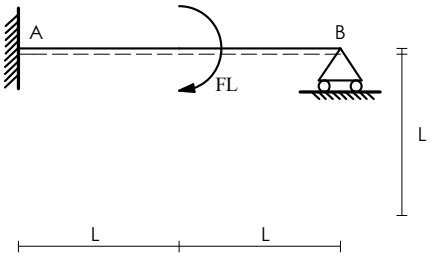
$M \Rightarrow$

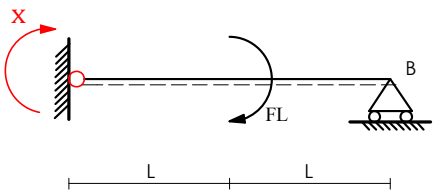


Esercizio N.6

Esercizio N.7

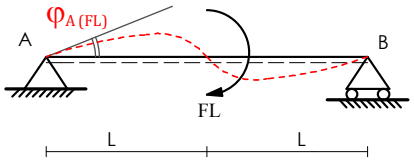






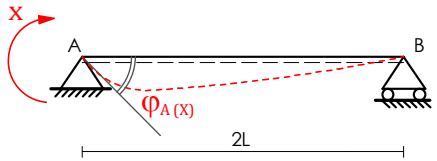
Eq. di Congruenza: $\boxed{\varphi_A = 0} \Rightarrow \varphi_{A\,(FL)} + \varphi_{A\,(X)} = 0$

$\boxed{\varphi_{A\,(FL)}} \Rightarrow$



$$\boxed{\varphi_{A\,(FL)} = \frac{FL(2L)}{24EJ}}$$

$\boxed{\varphi_{A\,(X)}} \Rightarrow$



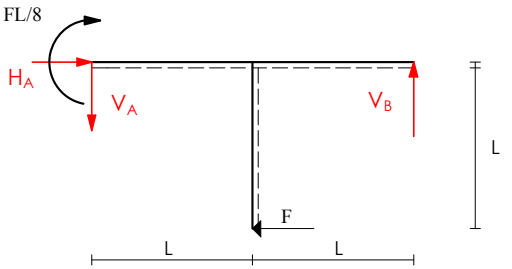
$$\boxed{\varphi_{A\,(X)} = -\frac{X2L}{3EJ}}$$

Eq. di Congruenza:

$$\varphi_{A\,(FL)} + \varphi_{A\,(X)} = 0 \Rightarrow \frac{FL(2L)}{24EJ} - \frac{X2L}{3EJ} = 0$$

$$X = \frac{2FL^2}{24EJ} \cdot \frac{3\,EJ}{2L} = \boxed{FL/8}$$

Esercizio N.7



Calcolo reazioni vincolari incognite:

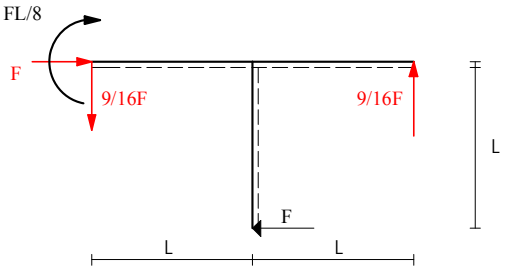
Eq. Ausiliaria : M_A

$\Sigma M_A = 0$
$-FL/8 + V_B \cdot 2L - F \cdot L = 0$
$\rightarrow \boxed{V_B = 9/16FL}$

$\Sigma V = 0$
$V_A - V_B = 0 \rightarrow \boxed{V_B = 9/16F}$

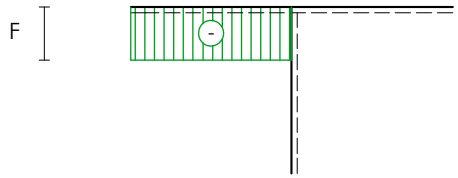
$\Sigma H = 0$
$H_A - F = 0 \rightarrow \boxed{H_A = F}$

Diagramma di corpo libero :

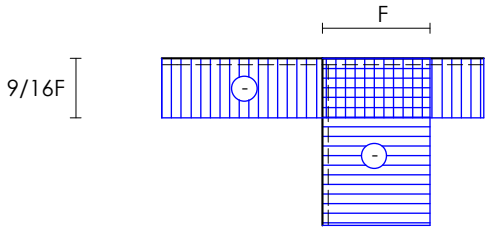


Diagrammi (N, T, M) :

$N \Rightarrow$

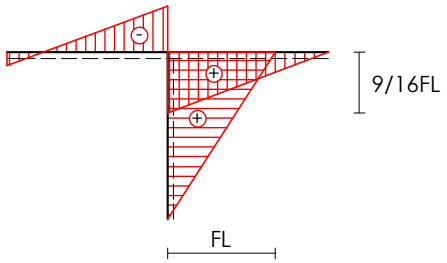


$T \Rightarrow$

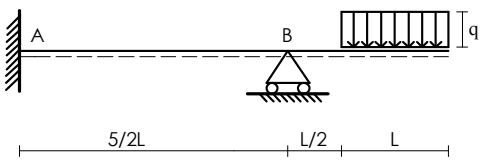


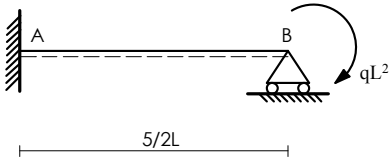
Esercizio N.7

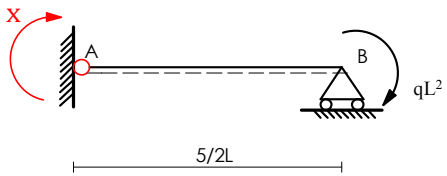
$M \Rightarrow$



Esercizio N.8

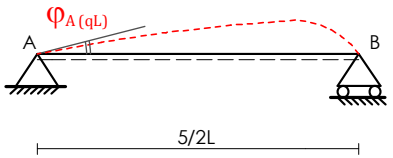






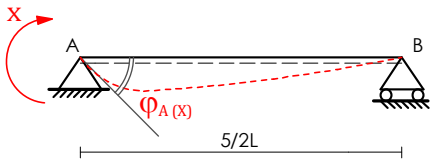
Eq. di Congruenza: $\boxed{\varphi_A = 0} \Rightarrow \varphi_{A\,(qL)} + \varphi_{A\,(X)} = 0$

$\boxed{\varphi_{A\,(qL)}} \Rightarrow$



$$\boxed{\varphi_{A\,(qL)} = \frac{qL^2(5/2L)}{6EJ}}$$

$\boxed{\varphi_{A\,(X)}} \Rightarrow$



$$\boxed{\varphi_{A\,(X)} = -\frac{X(5/2L)}{3EJ}}$$

Eq. di Congruenza:

$$\varphi_{A\,(qL)} + \varphi_{A\,(X)} = 0 \Rightarrow \frac{5/2qL^2}{6EJ} - \frac{XL5/2}{3EJ} = 0$$

$$X = \frac{5/2qL^2}{6EJ} \cdot \frac{3EJ}{5/2\,L} = \boxed{qL/2}$$

Esercizio N.8

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_A

$\Sigma M_A = 0$

$-qL/2 + V_B \cdot 5/2L - qL \cdot 7/2L = 0$

$\rightarrow \boxed{V_B = 8/5qL}$

$\Sigma V = 0$

$V_A - V_B = 0 \rightarrow \boxed{V_A = 3/5qL}$

$\Sigma H = 0$

$\boxed{H_A = 0}$

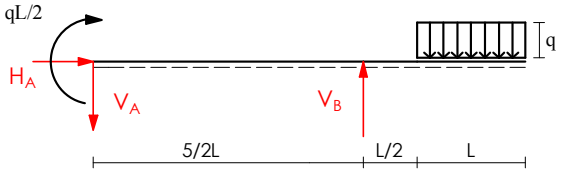
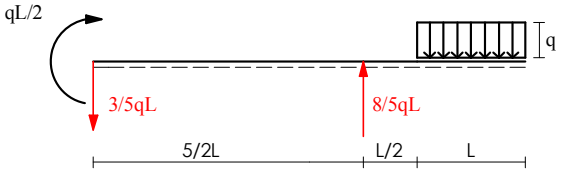


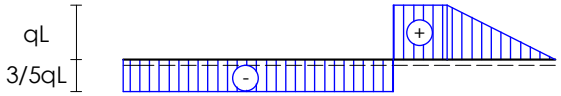
Diagramma di corpo libero :



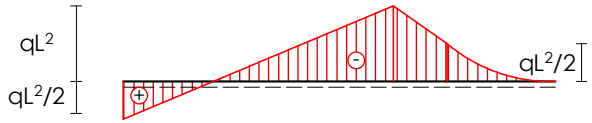
Diagrammi (N, T, M) :

$N \Rightarrow$

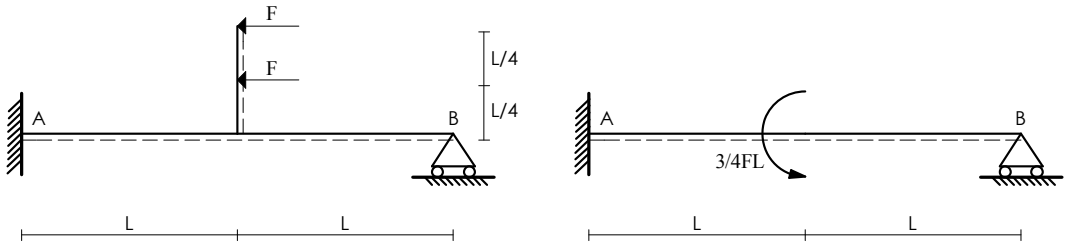
$T \Rightarrow$

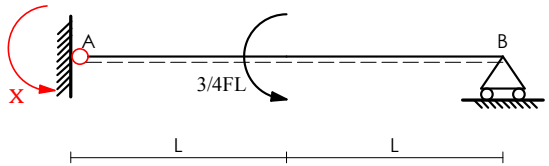


$M \Rightarrow$



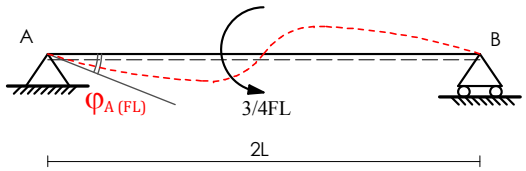
Esercizio N.9





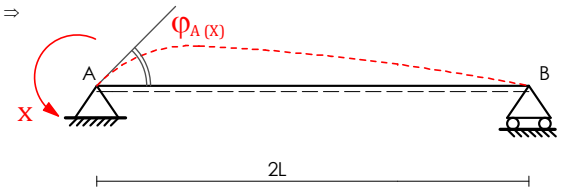
Eq. di Congruenza: $\boxed{\varphi_A = 0} \Rightarrow \varphi_{A\,(FL)} + \varphi_{A\,(X)} = 0$

$\boxed{\varphi_{A\,(FL)}} \Rightarrow$



$$\varphi_{A\,(FL)} = -\frac{3/4FL(2L)}{24EJ}$$

$\boxed{\varphi_{A\,(X)}} \Rightarrow$



$$\varphi_{A\,(X)} = -\frac{X(2L)}{3EJ}$$

Eq. di Congruenza:

$$\varphi_{A\,(FL)} + \varphi_{A\,(X)} = 0 \Rightarrow \frac{FL^2}{16EJ} - \frac{X2L}{3EJ} = 0$$

$$X = \frac{FL^2}{16EJ} \cdot \frac{3EJ}{2L} = \boxed{3/32}FL$$

Esercizio N.9

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_A

$\Sigma M_A = 0$

$-3/32F - V_B \cdot 2L + F \cdot L/4 + F \cdot L/4 = 0$

$\rightarrow \boxed{V_B = 27/64F}$

$\Sigma V = 0$

$V_A - V_B = 0 \rightarrow \boxed{V_A = 27/64F}$

$\Sigma H = 0$

$H_A - F - F = 0 \rightarrow \boxed{H_A = 2F}$

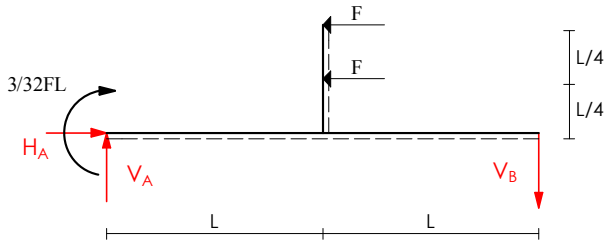
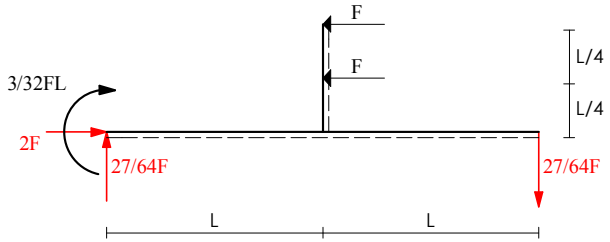
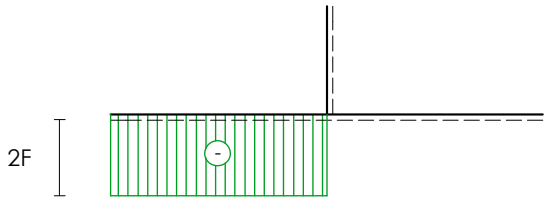


Diagramma di corpo libero :

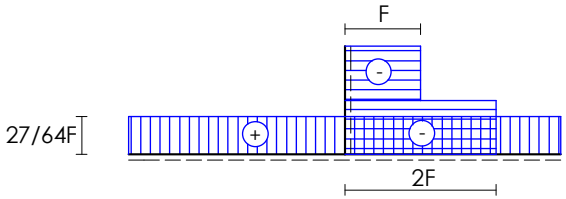


Diagrammi (N, T, M) :

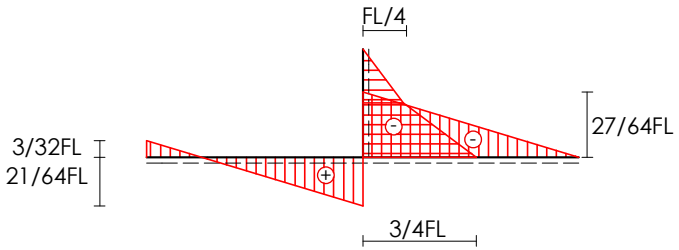
$N \Rightarrow$



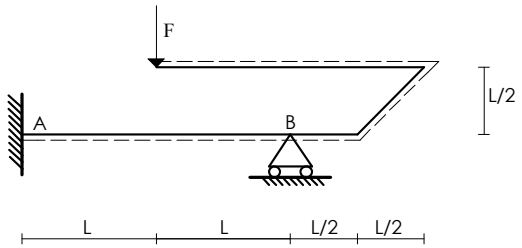
$T \Rightarrow$

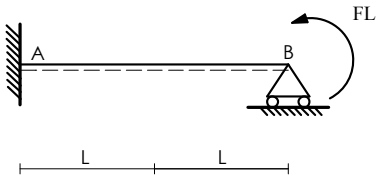


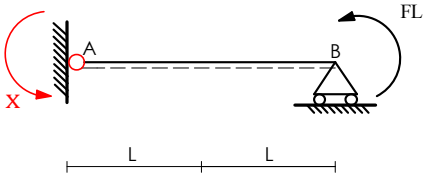
$M \Rightarrow$



Esercizio N.10

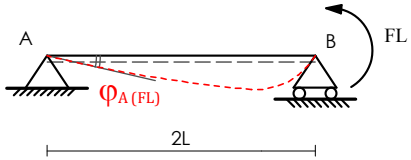






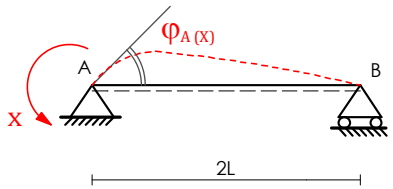
Eq. di Congruenza: $\boxed{\varphi_A = 0} \Rightarrow \varphi_{A\,(FL)} + \varphi_{A\,(X)} = 0$

$\boxed{\varphi_{A\,(FL)}} \Rightarrow$



$$\varphi_{A\,(FL)} = -\frac{FL(2L)}{6EJ}$$

$\boxed{\varphi_{A\,(X)}} \Rightarrow$



$$\varphi_{A\,(X)} = \frac{X(2L)}{3EJ}$$

Eq. di Congruenza:

$$\varphi_{A\,(FL)} + \varphi_{A\,(X)} = 0 \Rightarrow \frac{FL(2L)}{6EJ} - \frac{X(2L)}{3EJ} = 0$$

$$X = \frac{FL^2}{3EJ} \cdot \frac{3EJ}{2L} = \boxed{FL/2}$$

Esercizio N.10

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_A

$\Sigma M_A = 0$
$FL/2 + V_B \cdot 2L - F \cdot L = 0$
$\rightarrow \boxed{V_B = F/4}$

$\Sigma V = 0$
$V_A + V_B - F = 0 \rightarrow \boxed{V_A = 3/4F}$

$\Sigma H = 0$
$H_A = 0 \rightarrow \boxed{H_A = 0}$

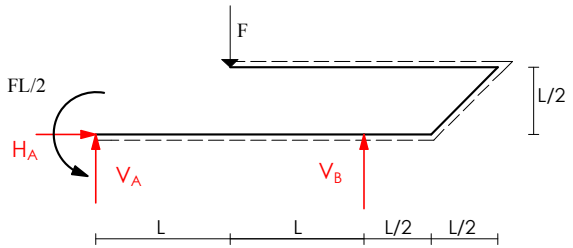
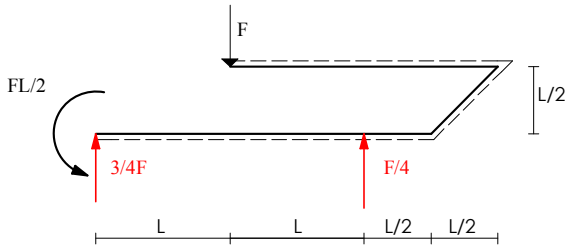
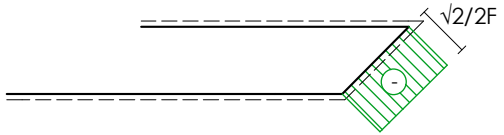


Diagramma di corpo libero :

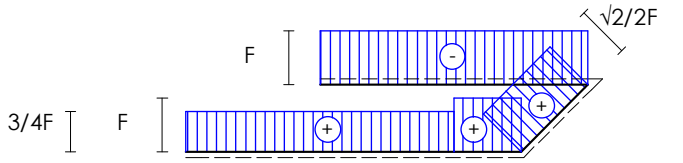


Diagrammi (N, T, M) :

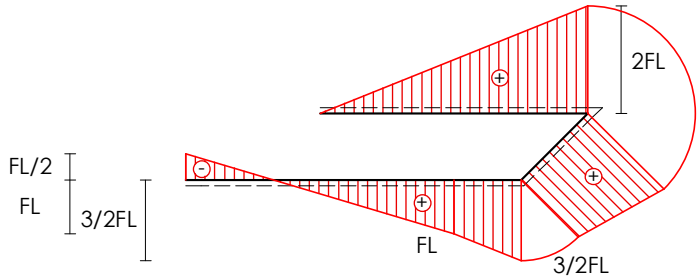
$N \Rightarrow$



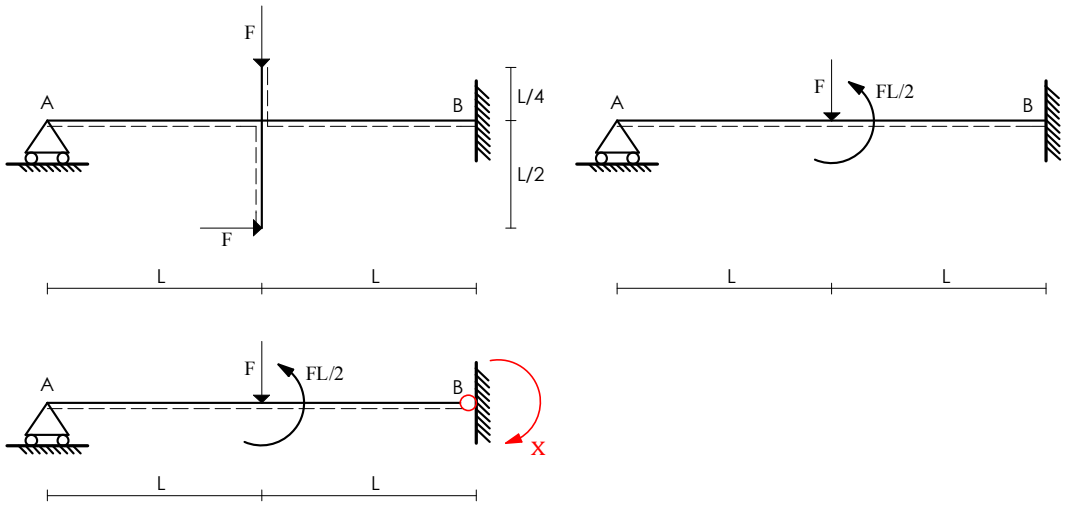
$T \Rightarrow$



$M \Rightarrow$

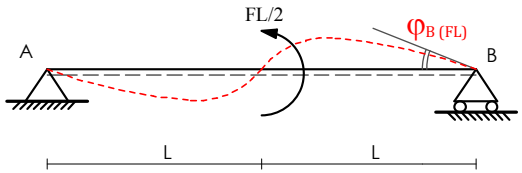


Esercizio N.11



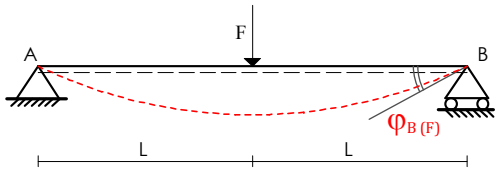
Eq. di Congruenza: $\boxed{\varphi_B = 0}$ $\Rightarrow$ $\varphi_{B\,(FL)} + \varphi_{B\,(F)} + \varphi_{B\,(X)} = 0$

$\boxed{\varphi_{B\,(FL)}}$ $\Rightarrow$



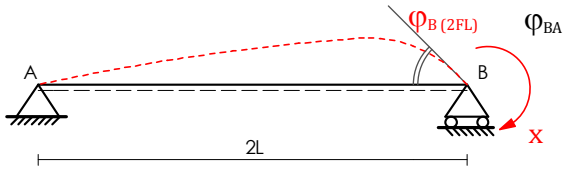
$$\boxed{\varphi_{B\,(FL)} = -\frac{FL/2(2L)}{24EJ}}$$

$\boxed{\varphi_{B\,(F)}}$ $\Rightarrow$



$$\boxed{\varphi_{B\,(F)} = \frac{F(2L)^2}{16EJ}}$$

$\boxed{\varphi_{B\,(X)}}$ $\Rightarrow$



$\varphi_{BA\,(X)} + \varphi_{BA\,(FL)} = \varphi_{BC\,(X)}$

$$\boxed{\varphi_{B\,(X)} = -\frac{X2L}{3EJ}}$$

Esercizio N.11

Eq. di Congruenza:

$$\varphi_{B\,(FL)} + \varphi_{B\,(F)} + \varphi_{B\,(X)} = 0 \quad \Rightarrow \quad -\frac{FL^2}{24EJ} + \frac{FL^2}{4EJ} - \frac{X2L}{3EJ} = 0$$

$$X = -\frac{5FL^2}{24EJ} \cdot \frac{3EJ}{2L} = \boxed{-5/16FL}$$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B

$\Sigma M_B = 0$

$5/16FL + V_A \cdot 2L - F \cdot L - F \cdot L/2 = 0$

$\rightarrow \boxed{V_A = 19/32F}$

$\Sigma V = 0$

$V_A + V_B - F = 0 \rightarrow \boxed{V_B = 13/32F}$

$\Sigma H = 0$

$H_B - F = 0 \rightarrow \boxed{H_B = F}$

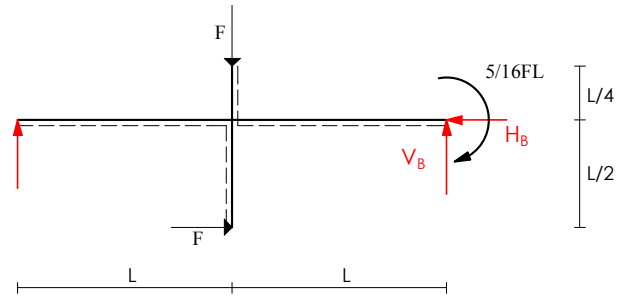
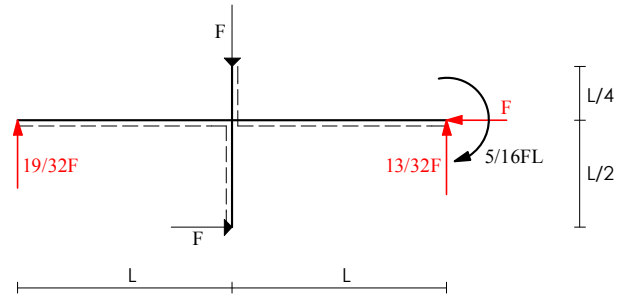
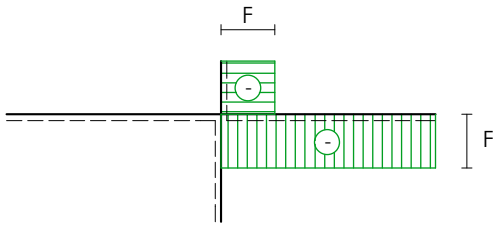


Diagramma di corpo libero :

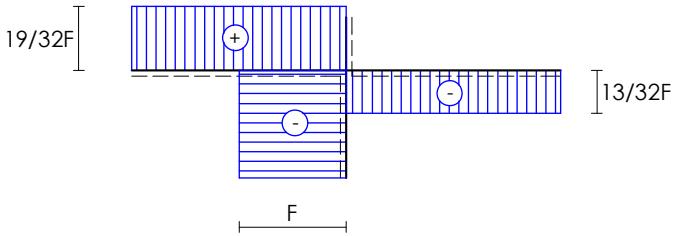


Diagrammi (N, T, M) :

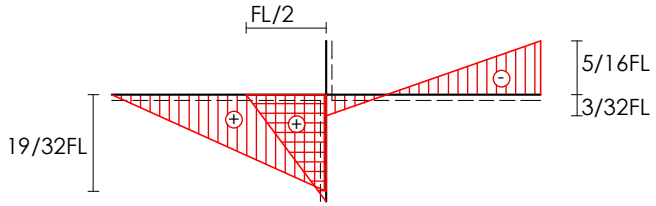


Esercizio N.11

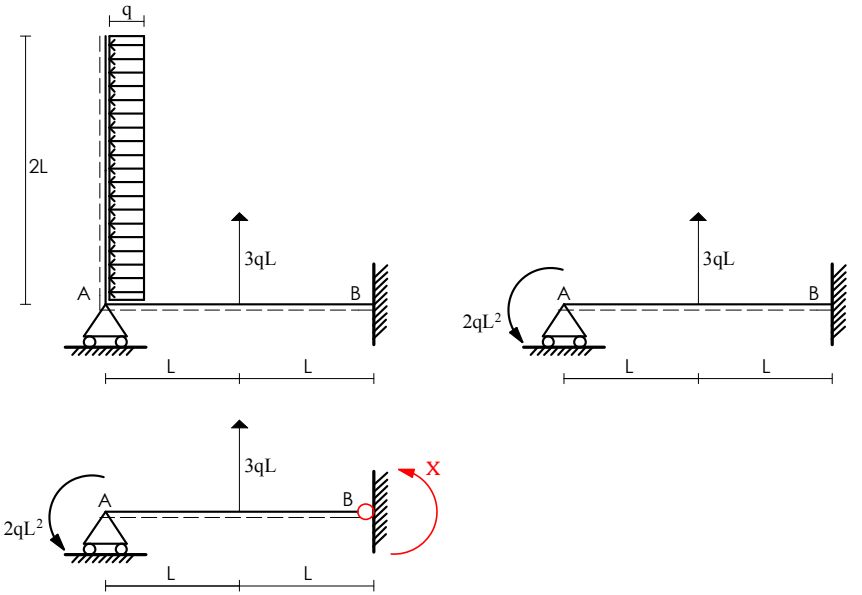
$T \Rightarrow$



$M \Rightarrow$

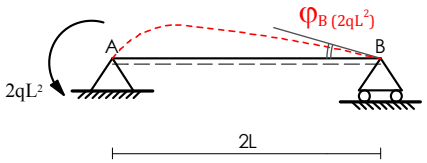


Esercizio N.12



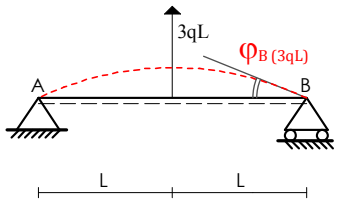
Eq. di Congruenza: $\boxed{\varphi_B = 0} \Rightarrow \varphi_{B\,(2qL^2)} + \varphi_{B\,(3qL)} + \varphi_{B\,(X)} = 0$

$\boxed{\varphi_{B\,(2qL^2)}} \Rightarrow$



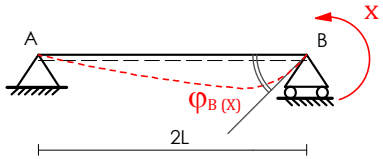
$$\varphi_{B\,(FL)} = -\frac{2qL^2(2L)}{6EJ}$$

$\boxed{\varphi_{B\,(3qL)}} \Rightarrow$



$$\varphi_{B\,(X)} = -\frac{3qL(2L)^2}{16EJ}$$

$\boxed{\varphi_{B\,(X)}} \Rightarrow$



$$\varphi_{B\,(X)} = \frac{X2L}{3EJ}$$

Esercizio N.12

Eq. di Congruenza:

$$\varphi_{B\,(2qL^2)} + \varphi_{B\,(3qL)} + \varphi_{B\,(X)} = 0 \quad \Rightarrow \quad -\frac{2qL^3}{3EJ} - \frac{3qL^3}{4EJ} + \frac{X2L}{3EJ} = 0$$

$$X = \frac{17FL^3}{12EJ} \cdot \frac{3EJ}{2L} = \boxed{17/8\ FL^2}$$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B

$\Sigma M_B = 0$
$17/8qL + V_A \cdot 2L - 2qL \cdot L + 3qL \cdot L = 0$
$\rightarrow \boxed{V_A = 9/16qL}$

$\Sigma V = 0$
$V_A - V_B + 3qL = 0 \rightarrow \boxed{V_B = 57/16qL}$

$\Sigma H = 0$
$H_B - 2qL = 0 \rightarrow \boxed{H_B = 2qL}$

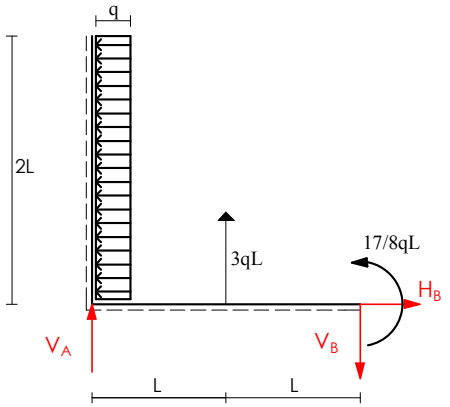
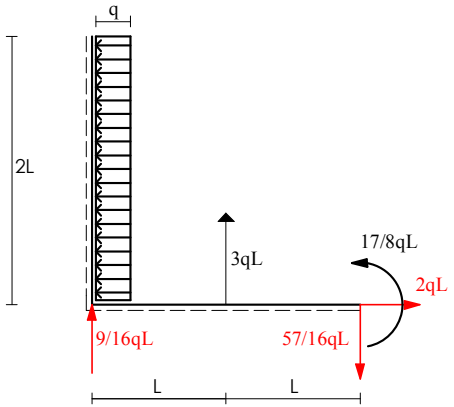


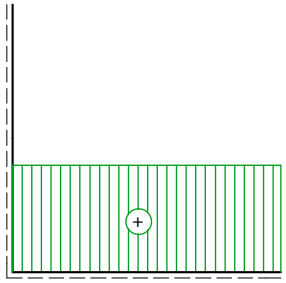
Diagramma di corpo libero :



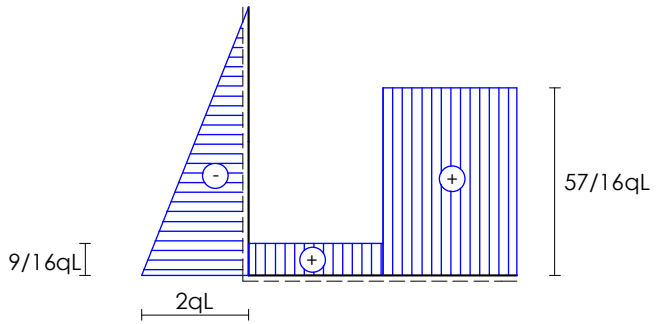
Esercizio N.12

Diagrammi (N, T, M) :

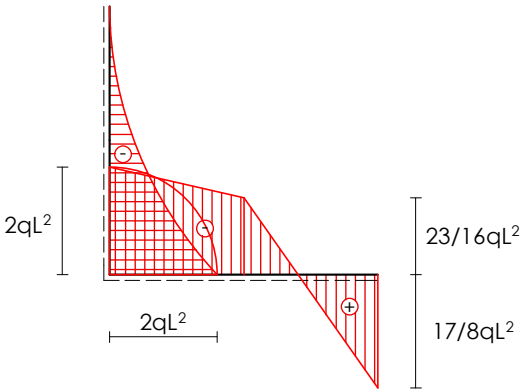
$N \Rightarrow$



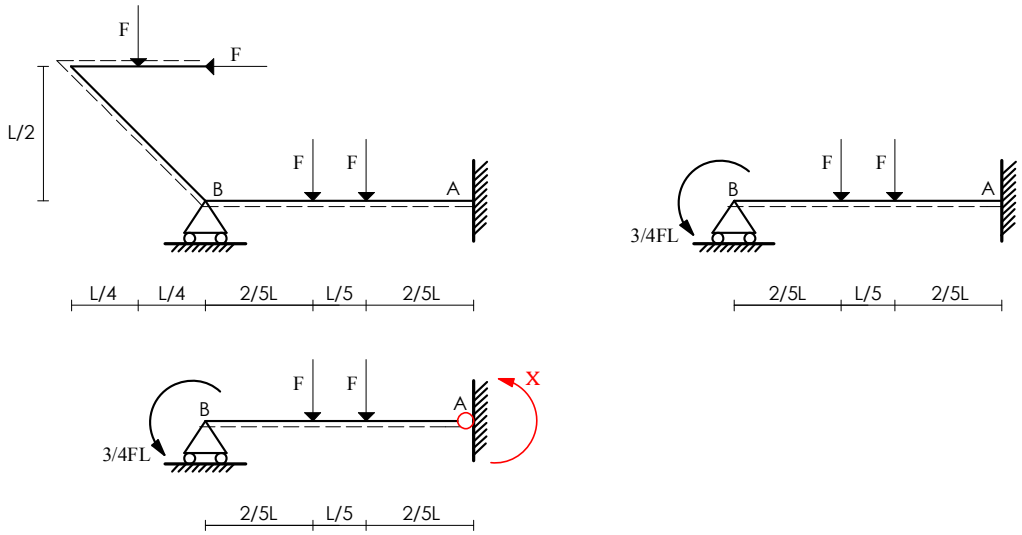
$T \Rightarrow$



$M \Rightarrow$

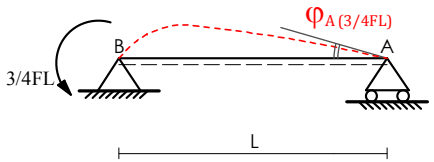


Esercizio N.13



Eq. di Congruenza: $\boxed{\varphi_A = 0} \Rightarrow \varphi_{A\,(3/4FL)} + \varphi_{A\,(F)} + \varphi_{A\,(X)} = 0$

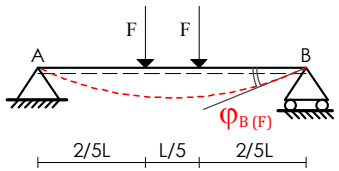
$\boxed{\varphi_{B\,(3/4FL)}} \Rightarrow$



$\varphi_{A\,(3/4FL)}$

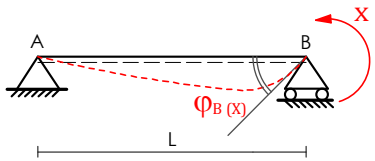
$$\boxed{\varphi_{B\,(FL)} = -\frac{3/4FL(L)}{6EJ}}$$

$\boxed{\varphi_{B\,(F)}} \Rightarrow$



$$\boxed{\varphi_{B\,(F)} = \frac{3FL^2}{25EJ}}$$

$\boxed{\varphi_{B\,(X)}} \Rightarrow$



$$\boxed{\varphi_{B\,(X)} = \frac{XL}{3EJ}}$$

Eq. di Congruenza:

$$\varphi_{A\,(3/4FL)} + \varphi_{A\,(F)} + \varphi_{A\,(X)} = 0 \Rightarrow -\frac{3/4FL^2}{3EJ} + \frac{3FL^2}{25EJ} + \frac{XL}{3EJ} = 0$$

$$X = \frac{1FL^3}{200EJ} \cdot \frac{3EJ}{L} = \boxed{3/200\ FL^2} = \boxed{0{,}015\ FL^2}$$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B

$\Sigma M_B = 0$

$0{,}015FL^2 - V_A \cdot L + F \cdot 2/5L + F \cdot 3/5L + F \cdot 5/4L + F \cdot L/2 = 0$

$\rightarrow \boxed{V_A = 2{,}765F}$

$\Sigma V = 0$

$V_A - V_B - F - F - F = 0 \rightarrow \boxed{V_B = 0{,}235F}$

$\Sigma H = 0$

$H_A - F = 0 \rightarrow \boxed{H_A = F}$

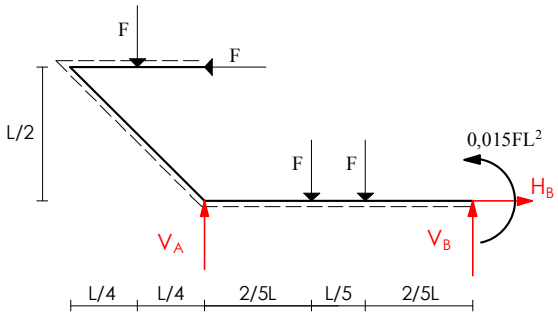
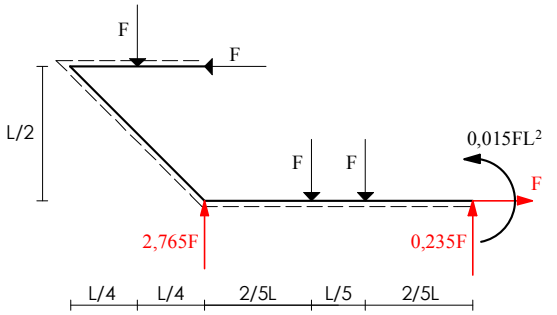
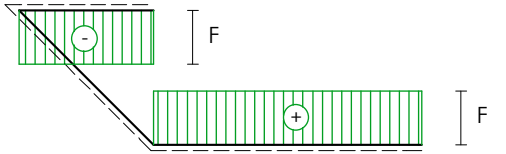


Diagramma di corpo libero :



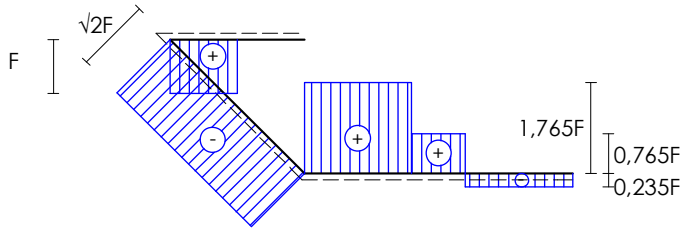
Diagrammi (N, T, M) :

$N \Rightarrow$

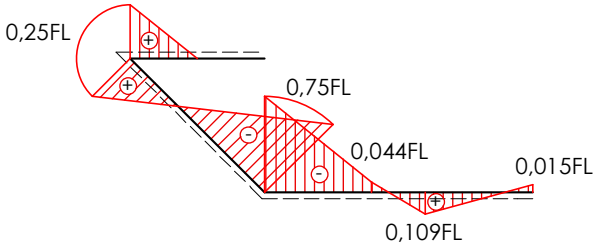


Esercizio N.13

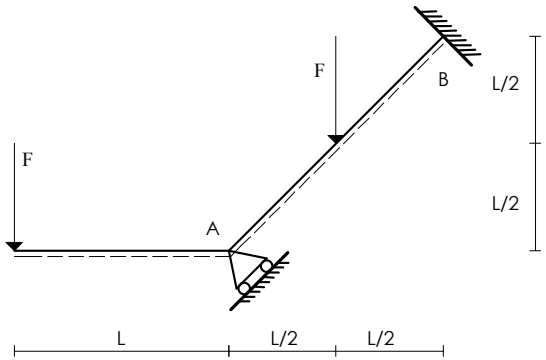
$T \Rightarrow$

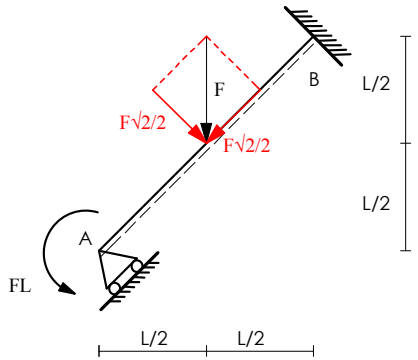


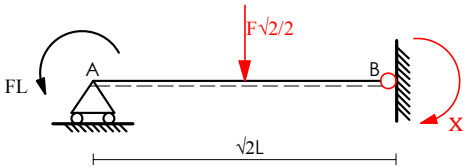
$M \Rightarrow$



Esercizio N.14

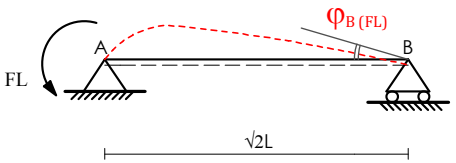






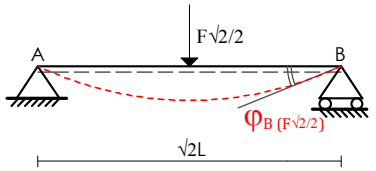
Eq. di Congruenza: $\boxed{\varphi_B = 0} \Rightarrow \varphi_{B\,(FL)} + \varphi_{B\,(F\sqrt{2}/2)} + \varphi_{B\,(X)} = 0$

$\boxed{\varphi_{B\,(FL)}} \Rightarrow$



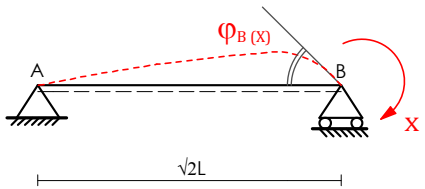
$$\boxed{\varphi_{B\,(FL)} = -\frac{FL(\sqrt{2}L)}{6EJ}}$$

$\boxed{\varphi_{B\,(F)}} \Rightarrow$



$$\boxed{\varphi_{B\,(F\sqrt{2}/2)} = \frac{F\sqrt{2}/2(\sqrt{2}L)^2}{16EJ}}$$

$\boxed{\varphi_{B\,(X)}} \Rightarrow$



$$\boxed{\varphi_{B\,(X)} = -\frac{X\sqrt{2}L}{3EJ}}$$

Eq. di Congruenza:

$$\varphi_{B\,(FL)} + \varphi_{B\,(F\sqrt{2}/2)} + \varphi_{B\,(X)} = 0 \quad \Rightarrow \quad -\frac{\sqrt{2}FL^2}{6EJ} + \frac{\sqrt{2}FL^2}{16EJ} - \frac{X\sqrt{2}L}{3EJ} = 0$$

$$X = -\frac{5\sqrt{2}FL^2}{48EJ} \cdot \frac{3EJ}{\sqrt{2}L} = \boxed{-5/16\ FL}$$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B

$\Sigma M_B = 0$

$0{,}015FL^2 - V_A \cdot L + F \cdot 2/5L + F \cdot 3/5L + F \cdot 5/4L + F \cdot L/2 = 0$

$\rightarrow \boxed{V_A = 2{,}765F}$

$\Sigma V = 0$

$V_A - V_B - F - F - F = 0 \rightarrow \boxed{V_B = 0{,}235F}$

$\Sigma H = 0$

$H_A - F = 0 \rightarrow \boxed{H_A = F}$

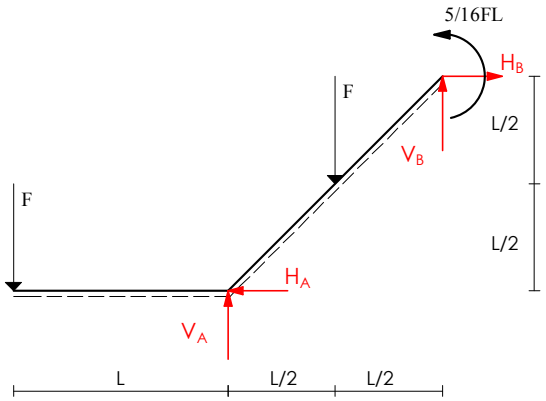
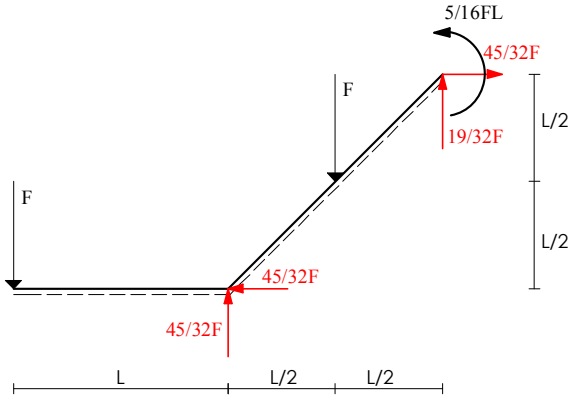
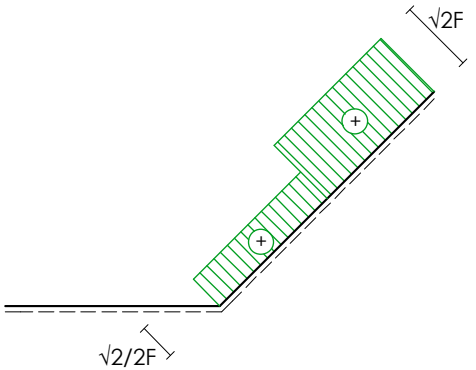


Diagramma di corpo libero :

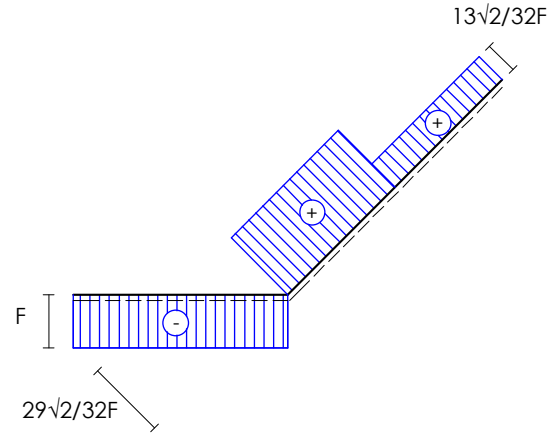


Diagrammi (N, T, M) :

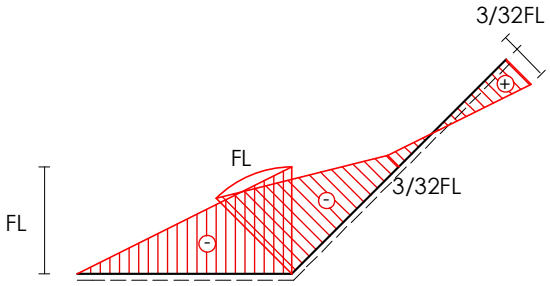
$N \Rightarrow$



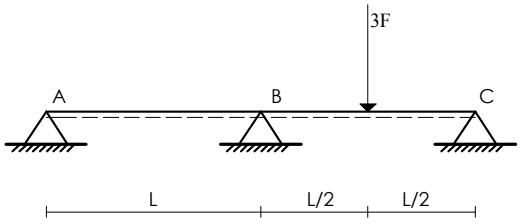
$T \Rightarrow$

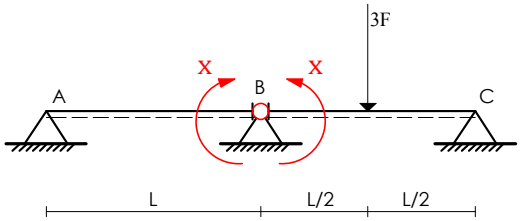


$M \Rightarrow$



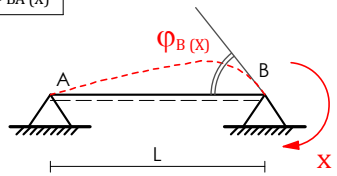
Esercizio N.15





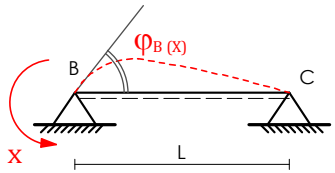
Eq. di Congruenza: $\Delta\varphi_B = 0 \Rightarrow \varphi_{BA} = \varphi_{BC} \Rightarrow \varphi_{BA\,(X)} = \varphi_{BC\,(X)} + \varphi_{BC\,(3F)}$

$\varphi_{BA\,(X)} \Rightarrow$



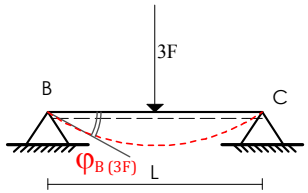
$$\varphi_{BA\,(X)} = -\frac{XL}{3\,EJ}$$

$\varphi_{BC\,(X)} \Rightarrow$



$$\varphi_{BC\,(X)} = \frac{XL}{3\,EJ}$$

$\varphi_{BC\,(X)} \Rightarrow$



$$\varphi_{BC\,(3F)} = -\frac{3FL^2}{16\,EJ}$$

Esercizio N.15

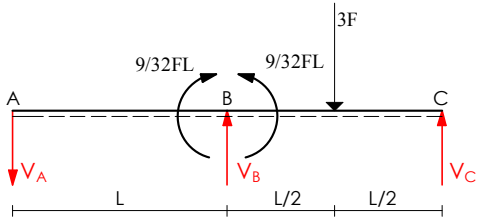
Eq. di Congruenza:

$$\varphi_{BA\,(X)} = \varphi_{BC\,(X)} + \varphi_{BC\,(3F)} \Rightarrow -\frac{XL}{3\,EJ} = \frac{XL}{3\,EJ} - \frac{3FL^2}{16\,EJ}$$

$$X = \frac{3FL^2}{16\,EJ} \cdot \frac{3\,EJ}{2L} = \boxed{9/32\,FL}$$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B



$\Sigma M_{BC} = 0$

$9/32FL + V_C \cdot L - 3F \cdot L/2 = 0 \rightarrow \boxed{V_C = 39/32FL}$

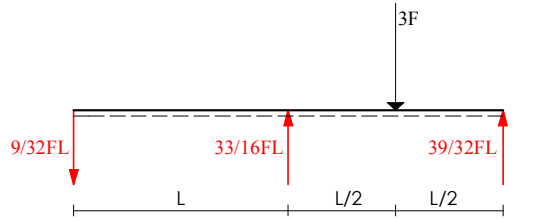
$\Sigma M_{BA} = 0$

$9/32FL - V_A \cdot L = 0 \rightarrow \boxed{V_A = 9/32FL}$

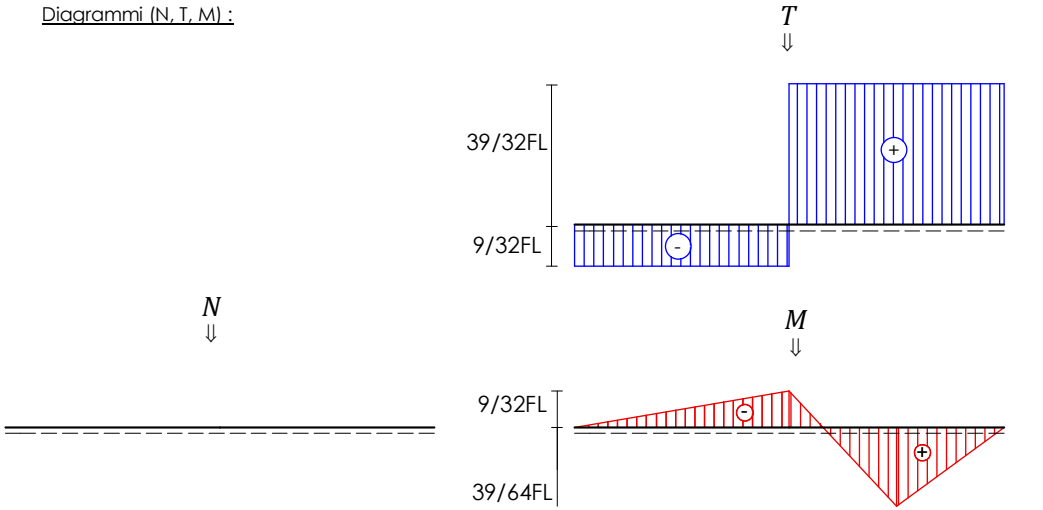
$\Sigma V = 0$

$- V_A + V_B + V_C - 3FL = 0 \rightarrow \boxed{V_B = 33/16FL}$

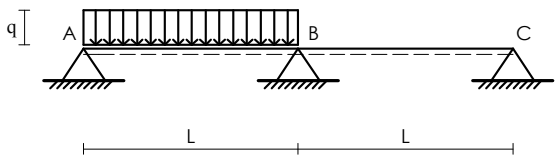
Diagramma di corpo libero :

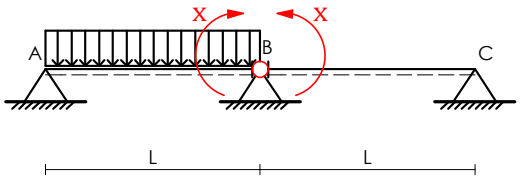


Diagrammi (N, T, M) :



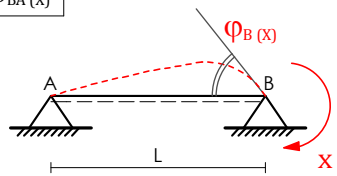
Esercizio N.16





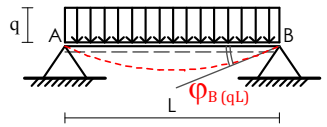
Eq. di Congruenza: $\boxed{\Delta\varphi_B = 0} \Rightarrow \boxed{\varphi_{BA} = \varphi_{BC}} \Rightarrow \varphi_{BA\,(X)} + \varphi_{BA\,(qL)} = \varphi_{BC\,(X)}$

$\boxed{\varphi_{BA\,(X)}} \Rightarrow$



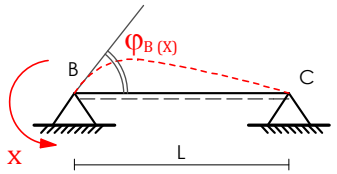
$$\boxed{\varphi_{BA\,(X)} = -\frac{XL}{3EJ}}$$

$\boxed{\varphi_{BA\,(qL)}} \Rightarrow$



$$\boxed{\varphi_{BA\,(qL)} = \frac{qL^3}{24EJ}}$$

$\boxed{\varphi_{BC\,(X)}} \Rightarrow$



$$\boxed{\varphi_{BC\,(X)} = \frac{XL}{3EJ}}$$

Esercizio N.16

Eq. di Congruenza:

$$\varphi_{BA\,(X)} + \varphi_{BA\,(qL)} = \varphi_{BC\,(X)} \Rightarrow -\frac{XL}{3EJ} + \frac{qL^3}{24EJ} = \frac{XL}{3EJ}$$

$$X = \frac{qL^3}{24EJ} \cdot \frac{3EJ}{2L} = \boxed{qL^2/16}$$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B

$\Sigma M_B = 0$
$-qL^2/16 - qL \cdot L/2 - V_A \cdot L = 0$
$\rightarrow \boxed{V_A = 7/16qL}$

$\Sigma M_B = 0$
$-qL^2/16 - V_C \cdot L = 0$
$\rightarrow \boxed{V_C = qL/16}$

$\Sigma V = 0$
$V_B + 7/16qL - qL - qL/16 = 0 \rightarrow \boxed{V_B = 5/8qL}$

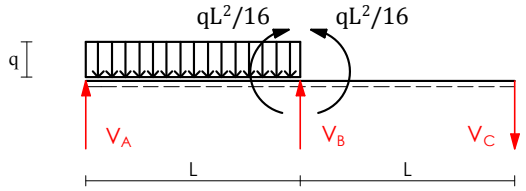
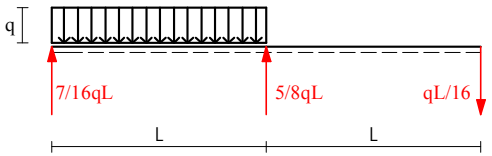
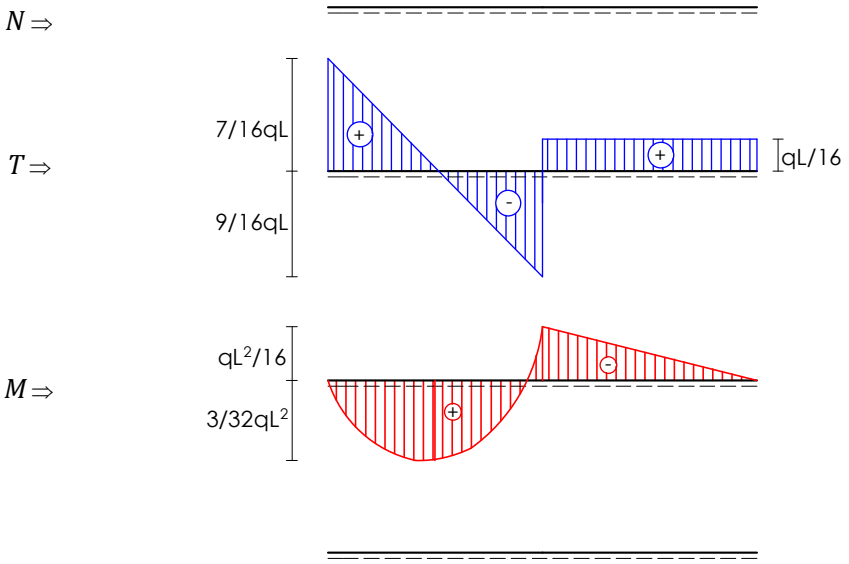
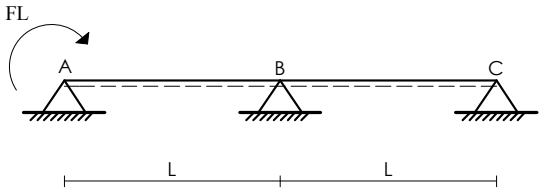


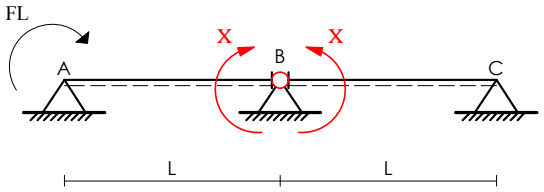
Diagramma di corpo libero :



Diagrammi (N, T, M) :

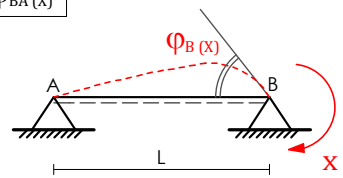






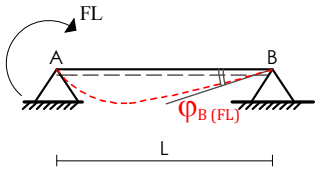
Eq. di Congruenza: $\boxed{\Delta\varphi_B = 0} \Rightarrow \boxed{\varphi_{BA} = \varphi_{BC}} \Rightarrow \varphi_{BA\,(X)} + \varphi_{BA\,(FL)} = \varphi_{BC\,(X)}$

$\boxed{\varphi_{BA\,(X)}} \Rightarrow$



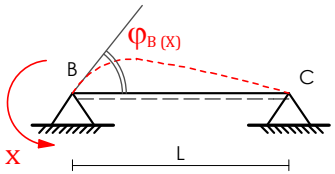
$$\varphi_{BA\,(X)} = -\frac{XL}{3EJ}$$

$\boxed{\varphi_{BA\,(FL)}} \Rightarrow$



$$\varphi_{BA\,(FL)} = \frac{FL(L)}{6EJ}$$

$\boxed{\varphi_{BC\,(X)}} \Rightarrow$



$$\varphi_{BC\,(X)} = \frac{XL}{3EJ}$$

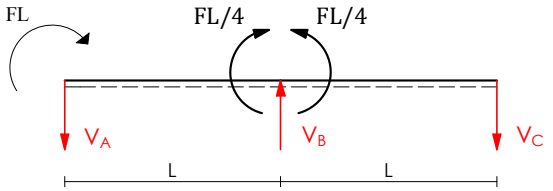
Eq. di Congruenza:

$$\varphi_{BA\,(X)} + \varphi_{BA\,(FL)} = \varphi_{BC\,(X)} \Rightarrow -\frac{XL}{3EJ} + \frac{FL^2}{6EJ} = \frac{XL}{3EJ}$$

$$X = \frac{FL^2}{6EJ} \cdot \frac{3EJ}{2L} = \boxed{FL/4}$$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B

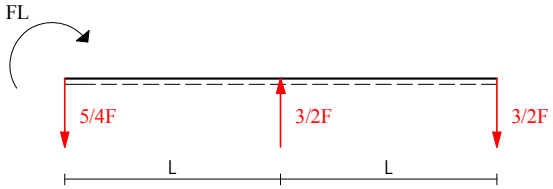


$\Sigma M_B = 0$
$FL/4 - V_C \cdot L = 0$
$\rightarrow \boxed{V_C = F/4}$

$\Sigma M_B = 0$
$- FL/4 - FL + V_A \cdot L = 0$
$\rightarrow \boxed{V_A = 5/4F}$

$\Sigma V = 0$
$V_B - 5/4F - F/4 = 0 \rightarrow \boxed{V_B = 3/2F}$

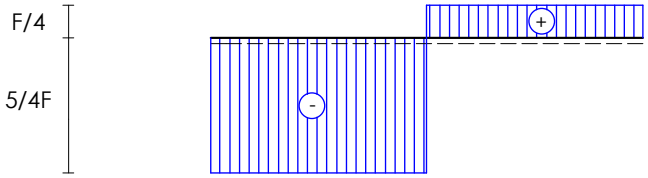
Diagramma di corpo libero :



Diagrammi (N, T, M) :

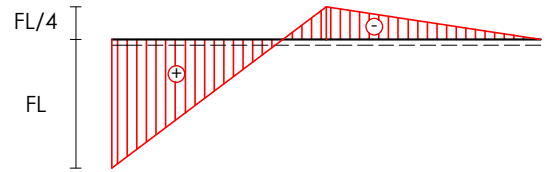
$N \Rightarrow$

$T \Rightarrow$

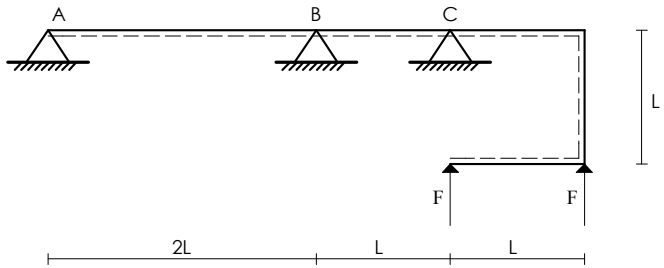


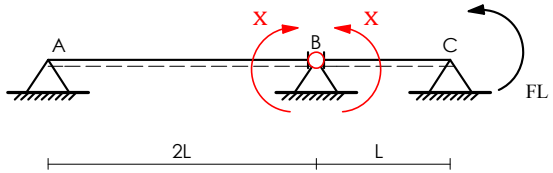
Esercizio N.17

$M \Rightarrow$



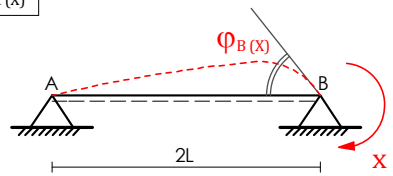
Esercizio N.18





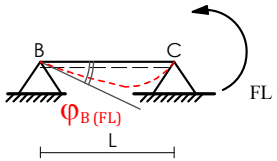
Eq. di Congruenza: $\boxed{\Delta\varphi_B = 0} \Rightarrow \boxed{\varphi_{BA} = \varphi_{BC}} \Rightarrow \varphi_{BA\,(X)} = \varphi_{BC\,(FL)} + \varphi_{BC\,(X)}$

$\boxed{\varphi_{BA\,(X)}} \Rightarrow$



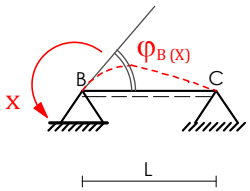
$$\varphi_{BA\,(X)} = -\frac{X2L}{3EJ}$$

$\boxed{\varphi_{BC\,(FL)}} \Rightarrow$



$$\varphi_{BC\,(FL)} = -\frac{FL(L)}{6EJ}$$

$\boxed{\varphi_{BC\,(X)}} \Rightarrow$



$$\varphi_{BC\,(X)} = \frac{XL}{3EJ}$$

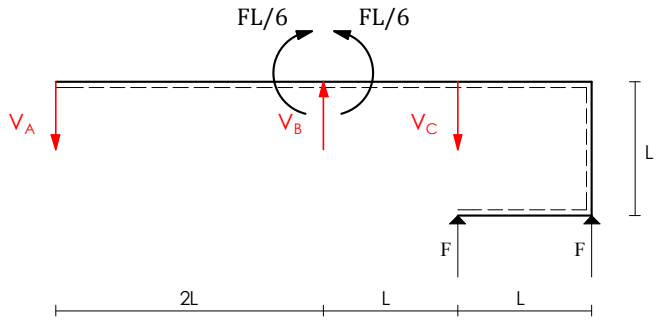
Eq. di Congruenza:

$$\varphi_{BA\,(X)} = \varphi_{BC\,(FL)} + \varphi_{BC\,(X)} \quad \Rightarrow \quad -\frac{X2L}{3EJ} = -\frac{FL^2}{6EJ} + \frac{XL}{3EJ}$$

$$X = \frac{FL^2}{6EJ} \cdot \frac{EJ}{L} = \boxed{FL/6}$$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B



$\Sigma M_B = 0$

$- FL/6 - V_C \cdot L + F \cdot L + F \cdot 2L = 0$

$\rightarrow \boxed{V_C = 19/6F}$

$\Sigma M_B = 0$

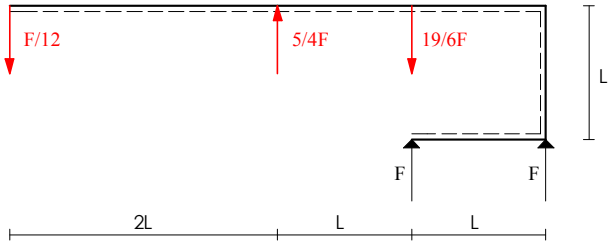
$- FL/6 - V_A \cdot 2L = 0 \rightarrow \boxed{V_A = F/12}$

$\Sigma V = 0$

$V_B - F/12 - 19/6F - F - F = 0$

$\rightarrow \boxed{V_B = 5/4F}$

Diagramma di corpo libero :

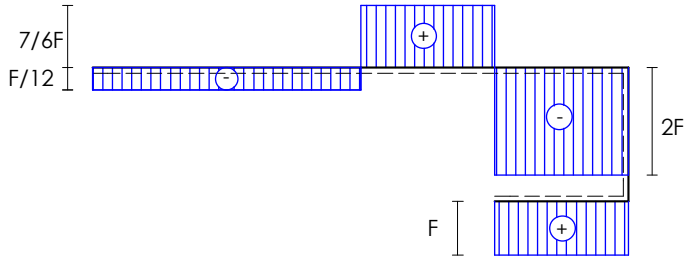


Diagrammi (N, T, M) :

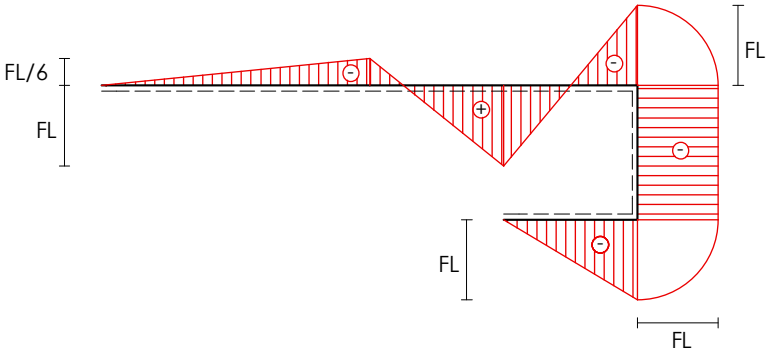
$N \Rightarrow$

Esercizio N.18

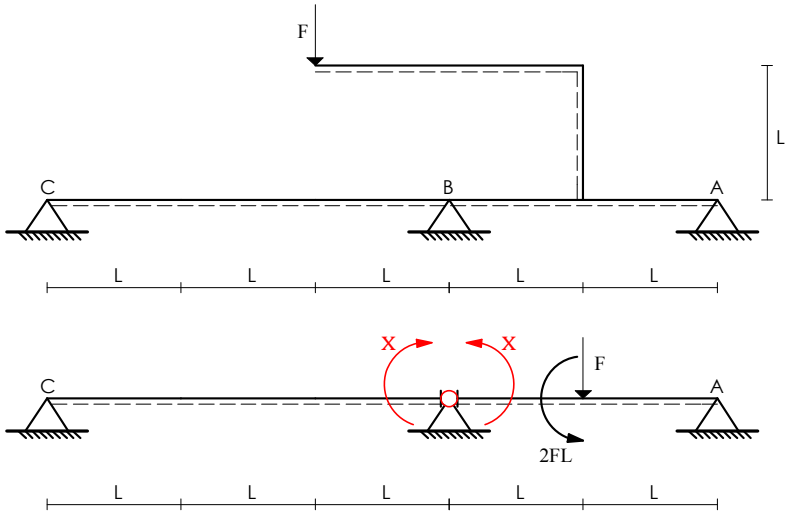
$T \Rightarrow$



$M \Rightarrow$

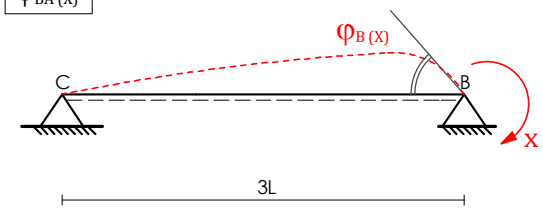


Esercizio N.19



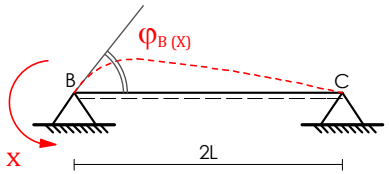
Eq. di Congruenza: $\Delta\varphi_B = 0 \Rightarrow \varphi_{BA} = \varphi_{BC} \Rightarrow \varphi_{BA\,(X)} = \varphi_{BC\,(X)} + \varphi_{BC\,(2FL)} + \varphi_{BC\,(F)}$

$\varphi_{BA\,(X)} \Rightarrow$



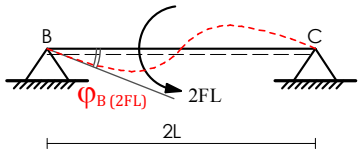
$$\varphi_{BA\,(X)} = -\frac{X3L}{3EJ}$$

$\varphi_{BC\,(X)} \Rightarrow$



$$\varphi_{BC\,(X)} = \frac{X2L}{3EJ}$$

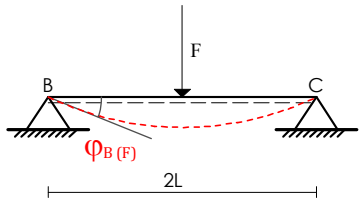
$\varphi_{BC\,(2FL)} \Rightarrow$



$$\varphi_{BC\,(2FL)} = -\frac{2FL(2L)}{24EJ}$$

Esercizio N.19

$\varphi_{BC\,(F)} \Rightarrow$



$$\varphi_{BC\,(F)} = -\frac{F(2L)^2}{16EJ}$$

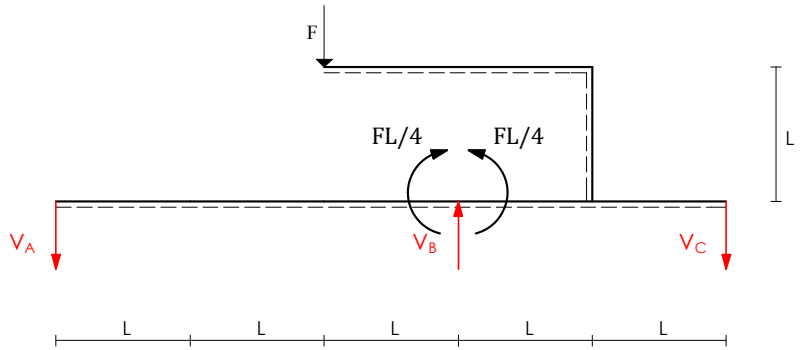
Eq. di Congruenza:

$$\varphi_{BA\,(X)} = \varphi_{BC\,(X)} + \varphi_{BC\,(2FL)} + \varphi_{BC\,(F)} \Rightarrow -\frac{XL}{EJ} = \frac{X2L}{3EJ} - \frac{FL^2}{6EJ} - \frac{FL^2}{4EJ}$$

$$\frac{XL}{EJ} + \frac{2XL}{3EJ} = \frac{5FL^2}{12EJ}$$

$$X = \frac{5FL^2}{12EJ} \cdot \frac{3EJ}{5L} = \boxed{FL/4}$$

Calcolo reazioni vincolari incognite:



Eq. Ausiliaria : M_B

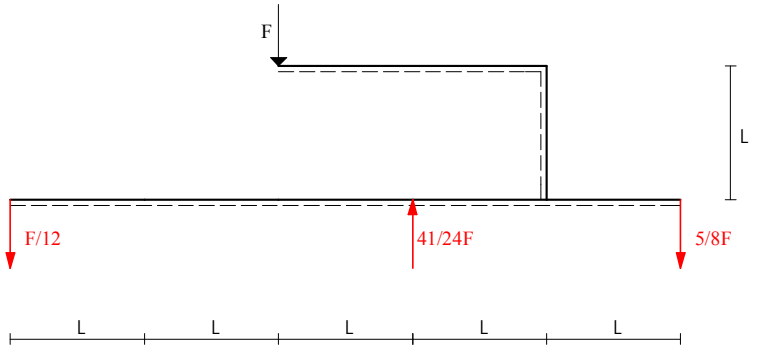
$\Sigma M_B = 0$
$FL/4 - V_C \cdot 2L + F \cdot L = 0$
$\rightarrow \boxed{V_C = 5/8F}$

$\Sigma V = 0$
$- V_B + F/12 + 5/8F + F = 0$
$\rightarrow \boxed{V_B = 41/24F}$

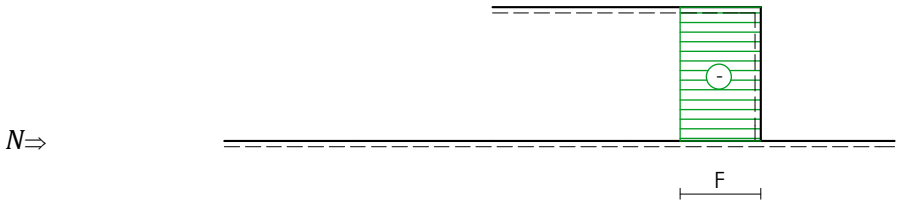
$\Sigma M_B = 0$
$- FL/4 + V_A \cdot 3L = 0 \rightarrow \boxed{V_A = F/12}$

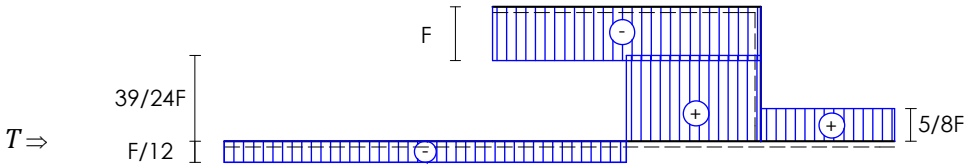
Esercizio N.19

Diagramma di corpo libero :

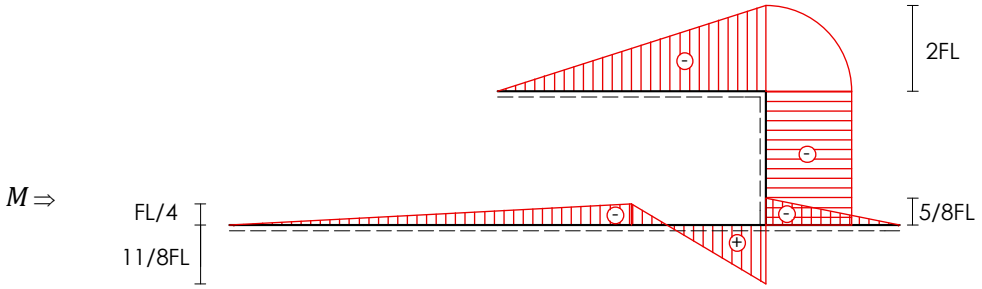


Diagrammi (N, T, M) :

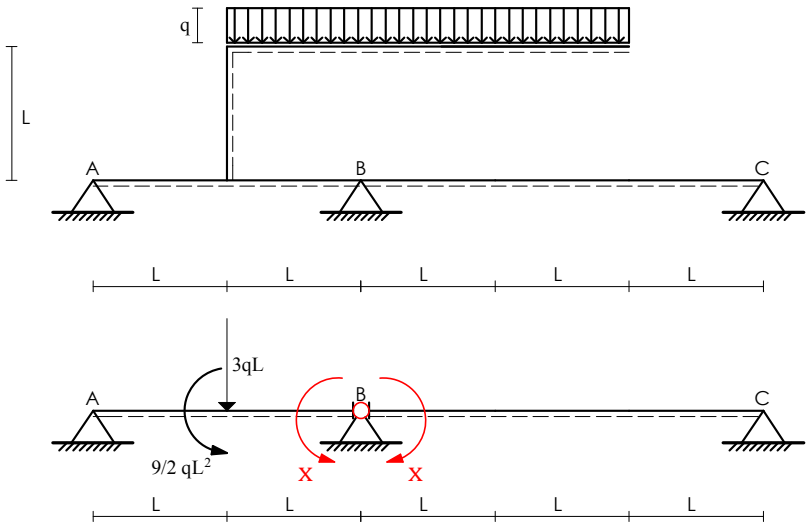




Esercizio N.19

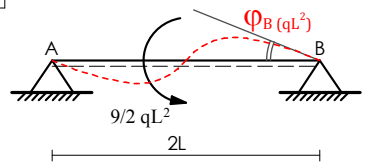


Esercizio N.20



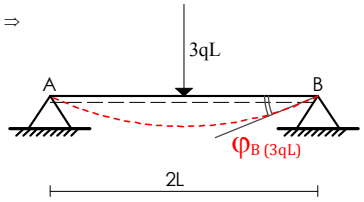
Eq. di Congruenza: $\boxed{\Delta\varphi_B = 0}$ ⇒ $\boxed{\varphi_{BA} = \varphi_{BC}}$ ⇒ $\varphi_{BA\,(qL^2)} + \varphi_{Ba\,(3qL)} + \varphi_{BA\,(X)} = \varphi_{BC\,(X)}$

$\boxed{\varphi_{BA\,(qL^2)}}$ ⇒



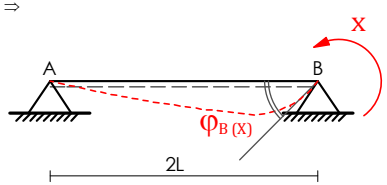
$$\boxed{\varphi_{BA\,(qL^2)} = -\frac{9/2qL^2\,(2L)}{24EJ}}$$

$\boxed{\varphi_{BA\,(3qL)}}$ ⇒



$$\boxed{\varphi_{BA\,(3qL)} = \frac{3qL(2L)^2}{16EJ}}$$

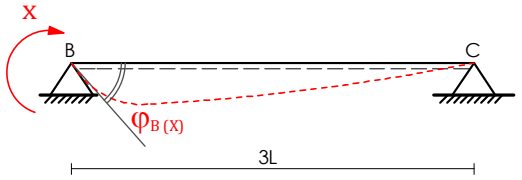
$\boxed{\varphi_{BA\,(X)}}$ ⇒



$$\boxed{\varphi_{BA\,(X)} = \frac{X(2L)}{3EJ}}$$

Esercizio N.20

$\boxed{\varphi_{BC\,(X)}}$ ⇒



$$\boxed{\varphi_{BC\,(X)} = -\frac{X(3L)}{3EJ}}$$

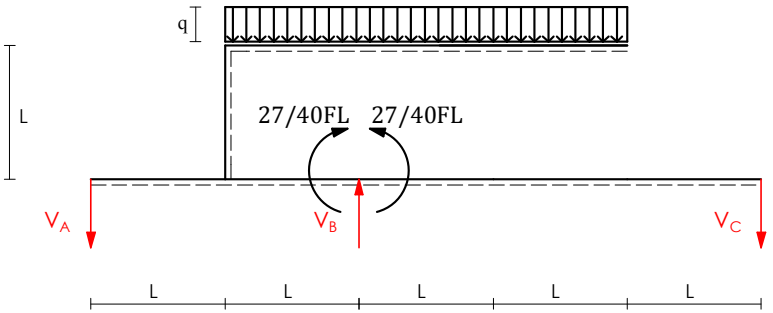
Eq. di Congruenza:

$$\varphi_{BA\,(qL^2)} + \varphi_{Ba\,(3qL)} + \varphi_{BA\,(X)} = \varphi_{BC\,(X)} \Rightarrow -\frac{3qL^3}{8EJ} + \frac{3qL^3}{4EJ} + \frac{X2L}{3EJ} = -\frac{XL}{EJ}$$

$$\frac{XL}{EJ} + \frac{2XL}{3EJ} = \frac{3qL^2}{8EJ}$$

$$X = \frac{3qL^2}{8EJ} \cdot \frac{3EJ}{5L} = \boxed{9/40\ qL}$$

Calcolo reazioni vincolari incognite:



Eq. Ausiliaria : M_B

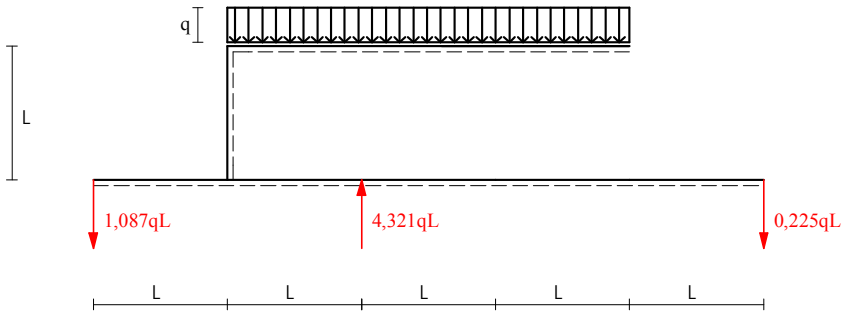
$\Sigma M_B = 0$
$27/40qL^2 - V_C \cdot 3L = 0$
$\rightarrow \boxed{V_C = 9/40qL = 0{,}225qL}$

$\Sigma V = 0$
$V_B - 87/80qL - 9/40qL - 3qL = 0$
$\rightarrow \boxed{V_B = 345/80qL = 4{,}321qL}$

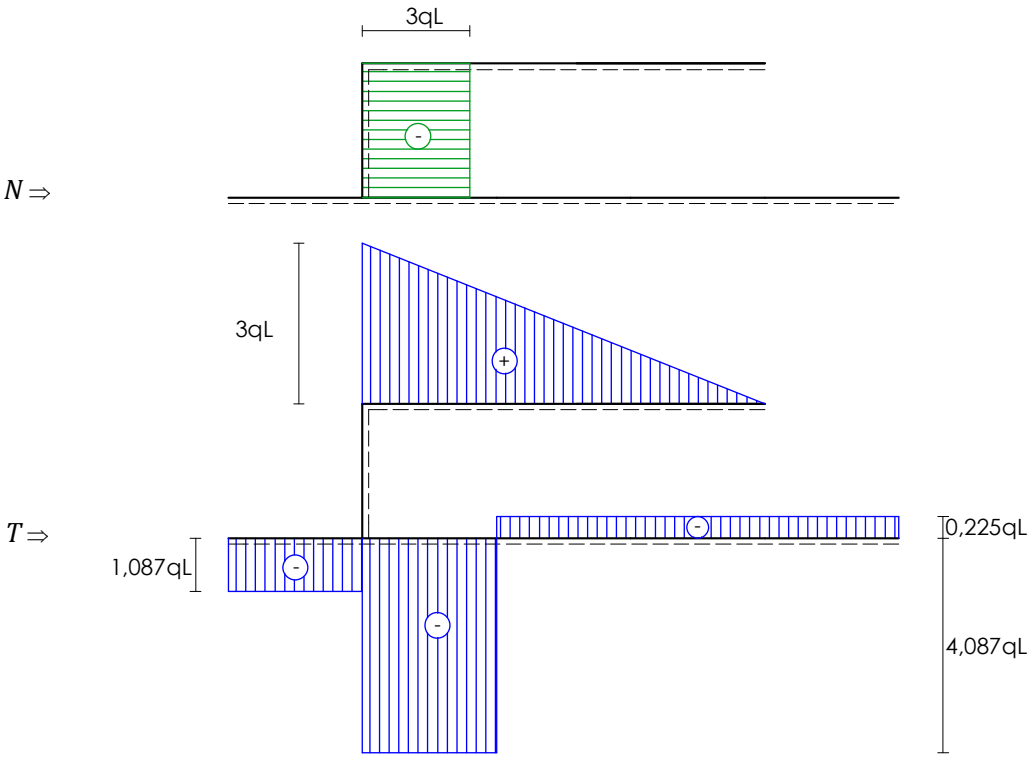
$\Sigma M_B = 0$
$3qL \cdot L + V_A \cdot 2L - 27/40qL^2 = 0$
$\rightarrow \boxed{V_A = 87/80qL = 1{,}087qL}$

Esercizio N.20

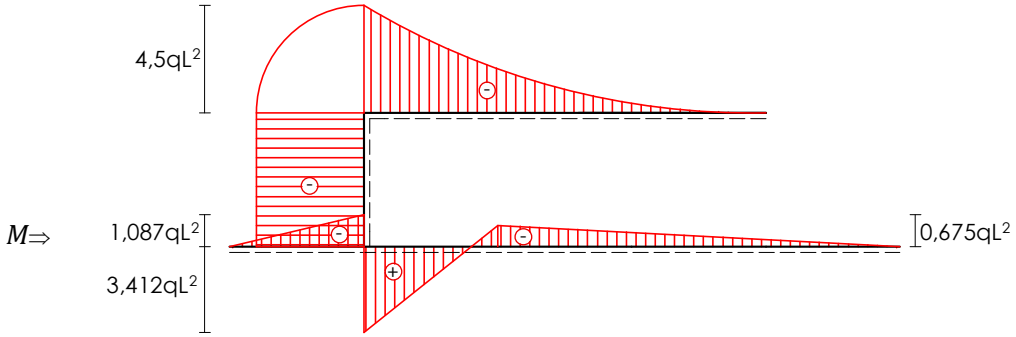
Diagramma di corpo libero :



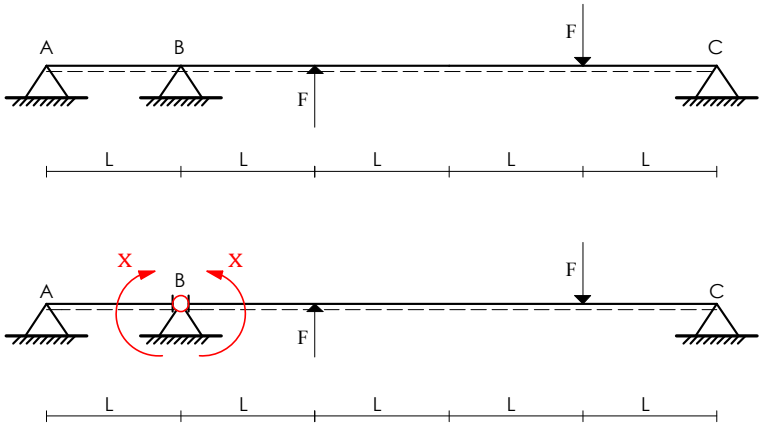
Diagrammi (N, T, M) :



Esercizio N.20

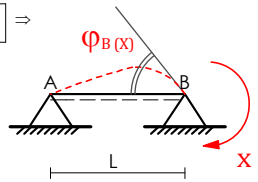


Esercizio N.21



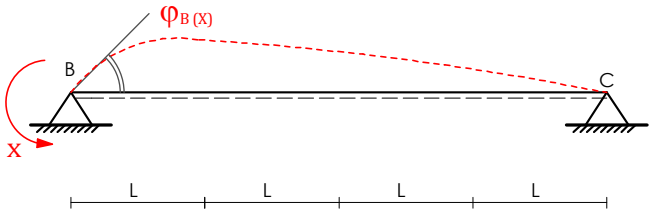
Eq. di Congruenza: $\boxed{\Delta\varphi_B = 0}$ ⇒ $\boxed{\varphi_{BA} = \varphi_{BC}}$ ⇒ $\varphi_{BA\,(X)} = \varphi_{BC\,(X)} + \varphi_{BC\,(F)} + \varphi_{BC\,(F')}$

$\boxed{\varphi_{BA\,(X)}}$ ⇒



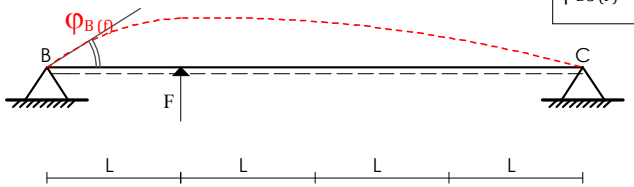
$$\boxed{\varphi_{BA\,(X)} = -\frac{XL}{3EJ}}$$

$\boxed{\varphi_{BC\,(X)}}$ ⇒



$$\boxed{\varphi_{BC\,(X)} = \frac{X(4L)}{3EJ}}$$

$\boxed{\varphi_{BC\,(F)}}$ ⇒

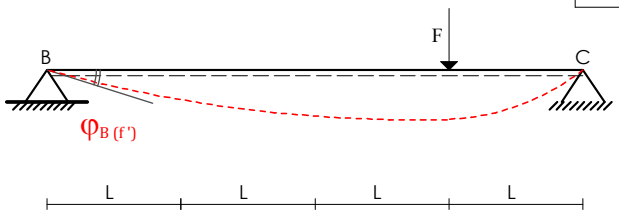


$$\boxed{\varphi_{BC\,(F)} = \frac{7F\,(4L)^2}{128EJL}}$$

Esercizio N.21

$\boxed{\varphi_{BC\,(F')}}$ ⇒

$$\boxed{\varphi_{BC\,(F')} = -\frac{5F(4L)^2}{128EJL}}$$



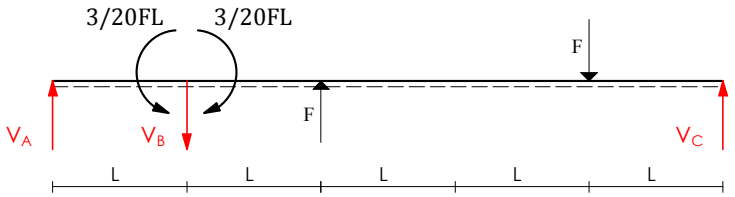
Eq. di Congruenza:

$$\varphi_{BA\,(X)} = \varphi_{BC\,(X)} + \varphi_{BC\,(F)} + \varphi_{BC\,(F')} \Rightarrow -\frac{XL}{3EJ} = \frac{4XL}{3EJ} + \frac{7FL^2}{8EJ} - \frac{5FL^2}{8EJ}$$

$$\frac{4XL}{3EJ} + \frac{XL}{3EJ} = \frac{5FL^2}{8EJ} - \frac{7FL^2}{8EJ}$$

$$X = -\frac{FL^2}{4EJ} \cdot \frac{3EJ}{5L} = \boxed{-3/20\ FL}$$

Calcolo reazioni vincolari incognite:



Eq. Ausiliaria : M_B

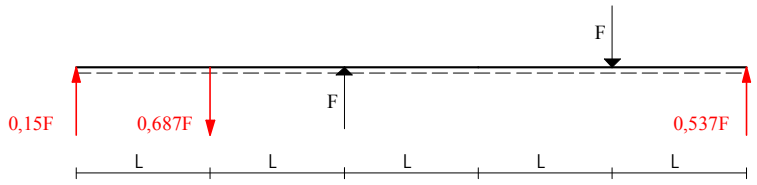
$\Sigma M_B = 0$
$-3/20FL + V_C \cdot 4L + F \cdot L - F \cdot 3L = 0$
$\rightarrow \boxed{V_C = 43/80F = 0{,}537F}$

$\Sigma V = 0$
$-V_B + 3/20F + 43/80F + F - F = 0$
$\rightarrow \boxed{V_B = 55/80F = 0{,}687F}$

$\Sigma M_B = 0$
$3/20FL - V_A \cdot L = 0 \rightarrow \boxed{V_A = 3/20F = 0{,}15F}$

Esercizio N.21

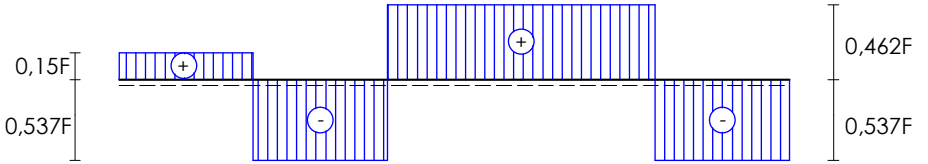
Diagramma di corpo libero :



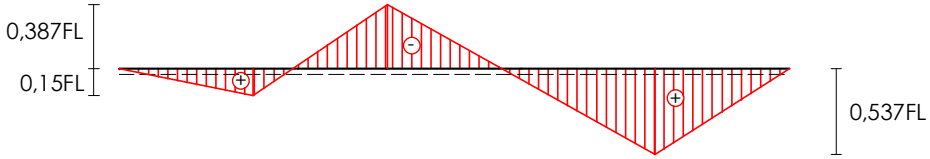
Diagrammi (N, T, M) :

$N \Rightarrow$

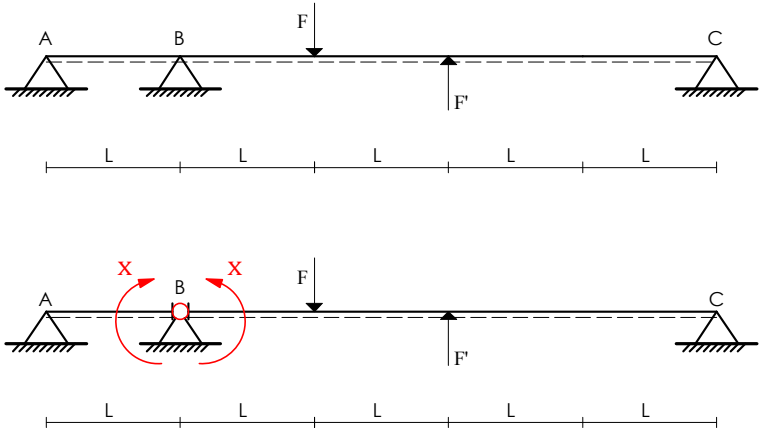
$T \Rightarrow$



$M \Rightarrow$

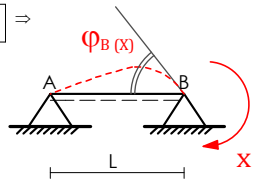


Esercizio N.22



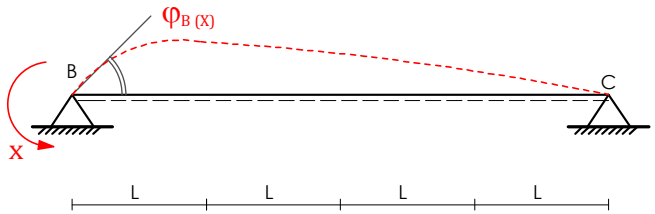
Eq. di Congruenza: $\boxed{\Delta\varphi_B = 0} \Rightarrow \boxed{\varphi_{BA} = \varphi_{BC}} \Rightarrow \varphi_{BA\,(X)} = \varphi_{BC\,(X)} + \varphi_{BC\,(F)} + \varphi_{BC\,(F')}$

$\boxed{\varphi_{BA\,(X)}} \Rightarrow$



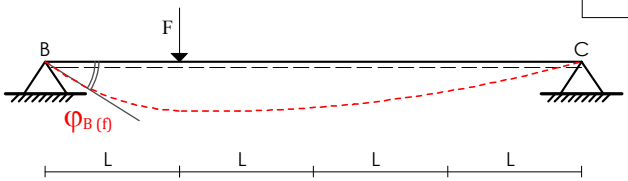
$$\boxed{\varphi_{BA\,(X)} = -\frac{XL}{3EJ}}$$

$\boxed{\varphi_{BC\,(X)}} \Rightarrow$



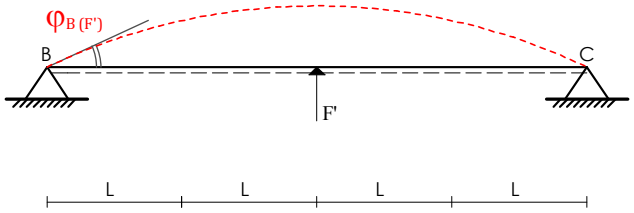
$$\boxed{\varphi_{BC\,(X)} = \frac{X(4L)}{3EJ}}$$

$\boxed{\varphi_{BC\,(F)}} \Rightarrow$



$$\boxed{\varphi_{BC\,(F)} = -\frac{7F(4L)^2}{128EJL}}$$

$\varphi_{BC\,(2F)} \Rightarrow$



$\varphi_{BC\,(F')} = \dfrac{F(4L)^2}{16EJ}$

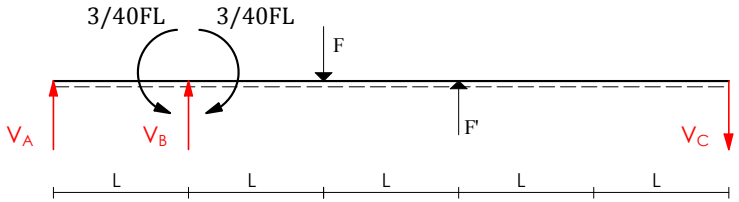
Eq. di Congruenza:

$$\varphi_{BA\,(X)} = \varphi_{BC\,(X)} + \varphi_{BC\,(F)} + \varphi_{BC\,(F')} \Rightarrow -\frac{XL}{3EJ} = \frac{4XL}{3EJ} - \frac{7FL^2}{8EJ} + \frac{FL^2}{EJ}$$

$$\frac{4XL}{3EJ} + \frac{XL}{3EJ} = \frac{FL^2}{EJ} - \frac{7FL^2}{8EJ}$$

$$X = -\frac{FL^2}{8EJ} \cdot \frac{3EJ}{5L} = \boxed{-3/40\ FL}$$

Calcolo reazioni vincolari incognite:



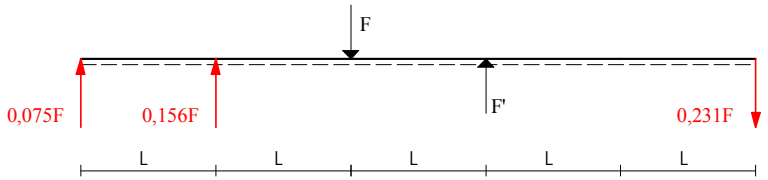
Eq. Ausiliaria : M_B

$\Sigma M_B = 0$
$-3/40FL + V_C \cdot 4L + F \cdot L + F \cdot L = 0$
$\rightarrow \boxed{V_C = 37/160F = 0{,}231F}$

$\Sigma V = 0$
$V_B + 3/40F - 37/160F + F + F = 0$
$\rightarrow \boxed{V_B = 25/160F = 0{,}156F}$

$\Sigma M_B = 0$
$3/40FL - V_A \cdot L = 0 \rightarrow \boxed{V_A = 3/40F = 0{,}075F}$

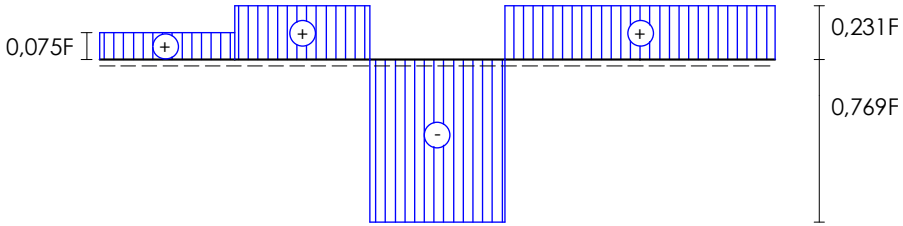
Diagramma di corpo libero :



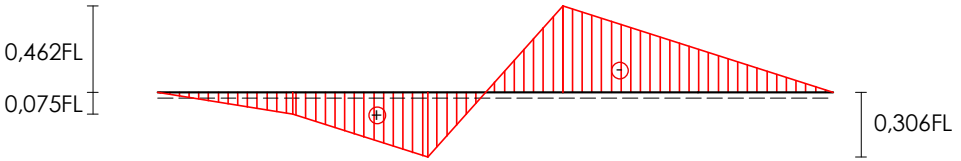
Diagrammi (N, T, M) :

$N \Rightarrow$

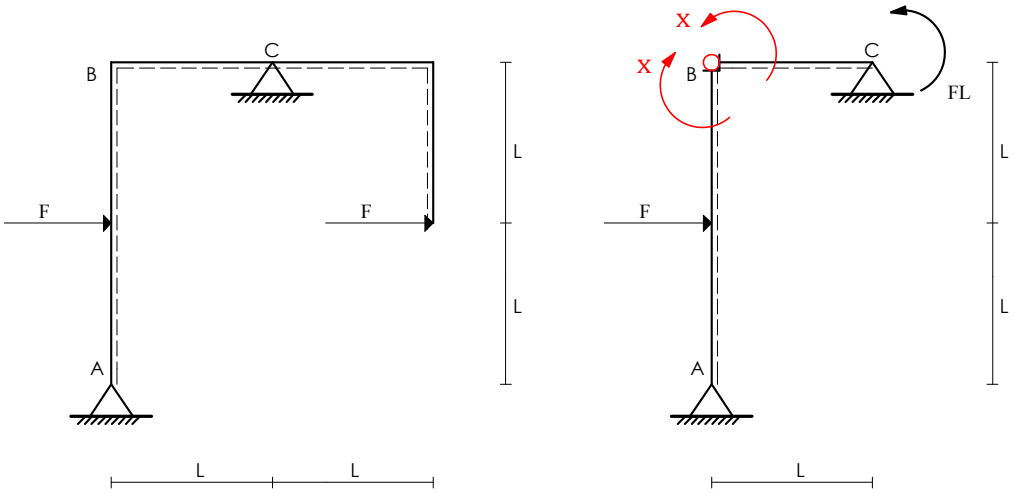
$T \Rightarrow$



$M \Rightarrow$

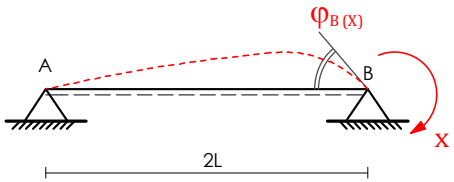


Esercizio N.23



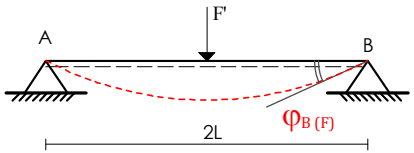
Eq. di Congruenza: $\Delta\varphi_B = 0 \Rightarrow \varphi_{BA} = \varphi_{BC} \Rightarrow \varphi_{BA\,(X)} + \varphi_{BA\,(F)} = \varphi_{BC\,(X)} + \varphi_{BC\,(FL)}$

$\varphi_{BA\,(X)} \Rightarrow$



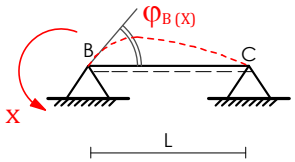
$$\varphi_{BA\,(X)} = -\frac{X2L}{3\,EJ}$$

$\varphi_{BA\,(F)} \Rightarrow$



$$\varphi_{BA\,(F)} = \frac{F(2L)^2}{16EJ}$$

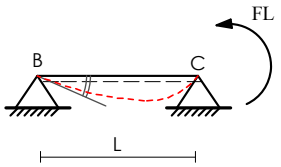
$\varphi_{BC\,(X)} \Rightarrow$



$$\varphi_{BC\,(X)} = \frac{XL}{3\,EJ}$$

Esercizio N.23

$\varphi_{BC\,(FL)} \Rightarrow$



$$\varphi_{BC\,(FL)} = -\frac{FL(L)}{6\,EJ}$$

Eq. di Congruenza:

$$\varphi_{BA\,(X)} + \varphi_{BA\,(F)} = \varphi_{BC\,(X)} + \varphi_{BC\,(FL)} \Rightarrow -\frac{2XL}{3\,EJ} + \frac{FL^2}{4\,EJ} = \frac{XL}{3\,EJ} - \frac{FL^2}{6\,EJ}$$

$$X = \frac{5FL^2}{12\,EJ} \cdot \frac{EJ}{L} = \boxed{5/12FL}$$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B

$\Sigma M_B = 0$

$0{,}015FL^2 - V_A \cdot L + F \cdot 2/5L + F \cdot 3/5L + F \cdot 5/4L + F \cdot L/2 = 0$

$\rightarrow \boxed{V_A = 2{,}765F}$

$\Sigma V = 0$

$V_A - V_B - F - F - F = 0 \rightarrow \boxed{V_B = 0{,}235F}$

$\Sigma H = 0$

$H_A - F = 0 \rightarrow \boxed{H_A = F}$

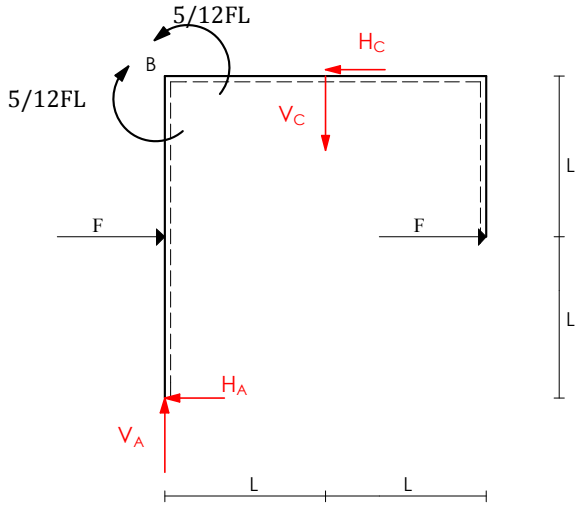
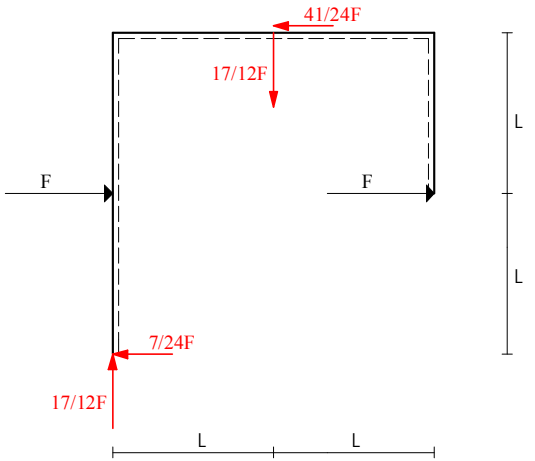
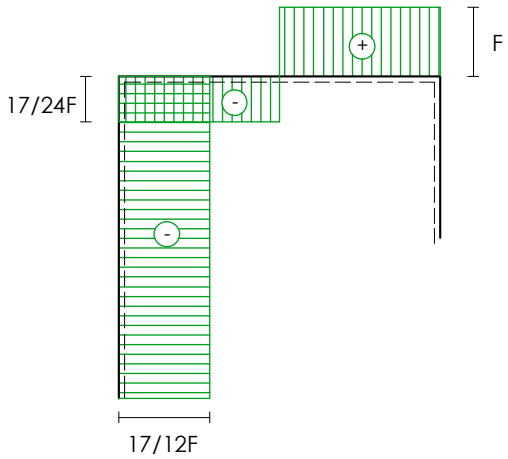


Diagramma di corpo libero :

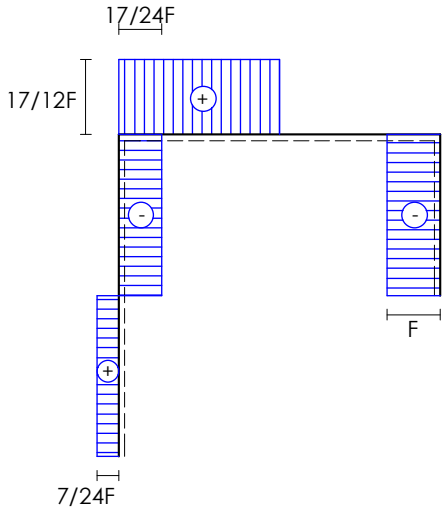


Diagrammi (N, T, M) :

$N \Rightarrow$

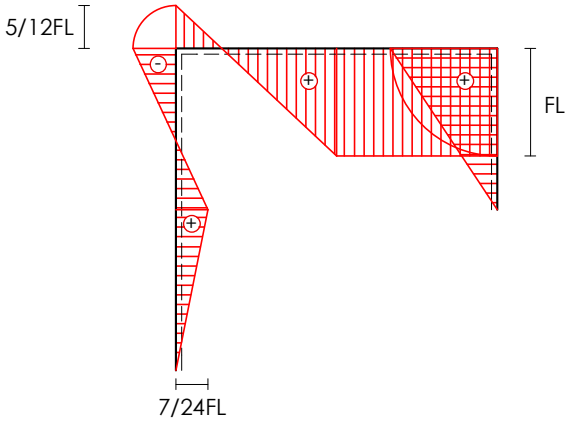


$T \Rightarrow$

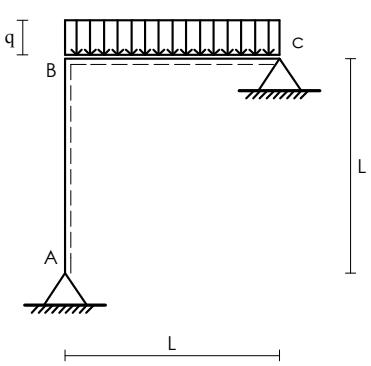


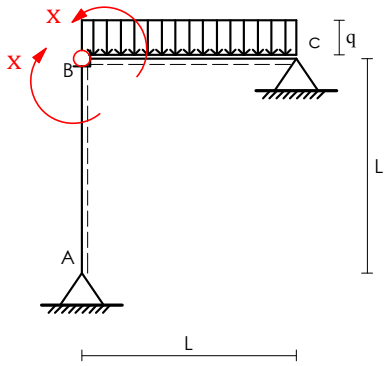
Esercizio N.23

$M \Rightarrow$



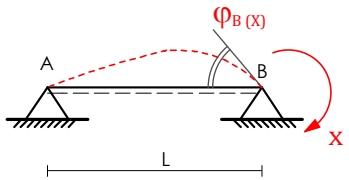
Esercizio N.24





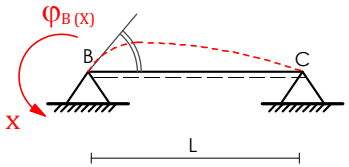
Eq. di Congruenza: $\Delta\varphi_B = 0 \Rightarrow \varphi_{BA} = \varphi_{BC} \Rightarrow \varphi_{BA\,(X)} = \varphi_{BC\,(X)} + \varphi_{BC\,(qL)}$

$\varphi_{BA\,(X)} \Rightarrow$



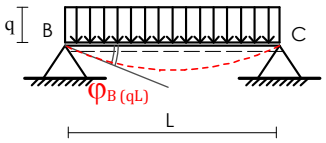
$$\varphi_{BA\,(X)} = -\frac{XL}{3\,EJ}$$

$\varphi_{BC\,(X)} \Rightarrow$



$$\varphi_{BC\,(X)} = \frac{XL}{3\,EJ}$$

$\varphi_{BC\,(qL)} \Rightarrow$



$$\varphi_{BC\,(qL)} = -\frac{qL^3}{24\,EJ}$$

Eq. di Congruenza:

$$\varphi_{BA\,(X)} = \varphi_{BC\,(X)} + \varphi_{BC\,(qL)} \Rightarrow -\frac{XL}{3\,EJ} = \frac{XL}{3\,EJ} - \frac{qL^3}{24\,EJ}$$

$$X = \frac{qL^3}{24\,EJ} \cdot \frac{3\,EJ}{2L} = \boxed{qL^2/16}$$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B

$\Sigma M_B = 0$
$-qL \cdot L/2 + V_c \cdot L + qL^2/16 = 0$
$\rightarrow \boxed{V_C = 7/16qL}$

$\Sigma V = 0$
$V_A + 7/16qL - qL = 0 \rightarrow \boxed{V_A = 9/16qL}$

$\Sigma M_B = 0$
$- qL^2/16 + H_A \cdot L = 0 \rightarrow \boxed{H_A = qL/16}$

$\Sigma H = 0$
$H_C - qL/16 = 0 \rightarrow \boxed{H_C = qL/16}$

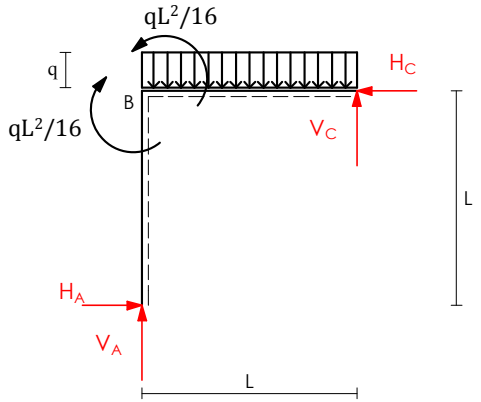
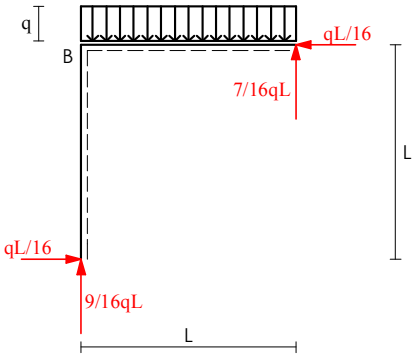


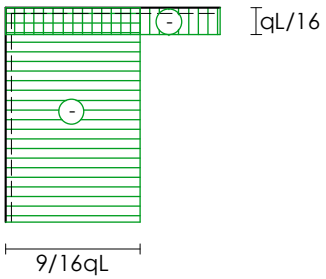
Diagramma di corpo libero :



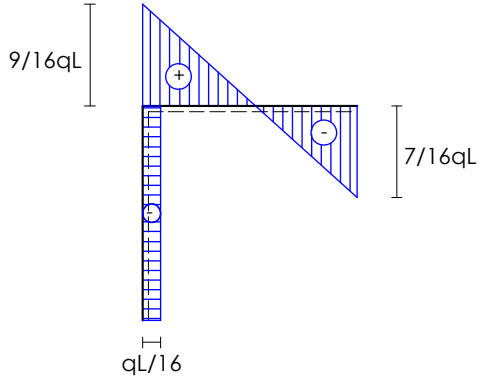
Esercizio N.24

Diagrammi (N, T, M) :

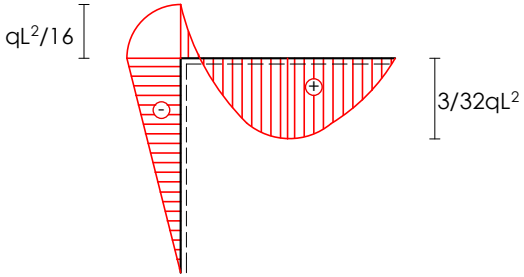
$N \Rightarrow$



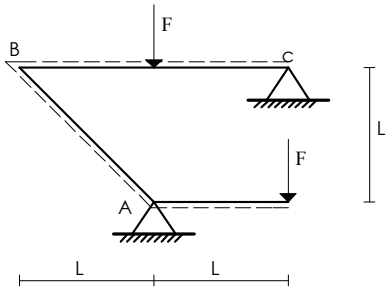
$T \Rightarrow$

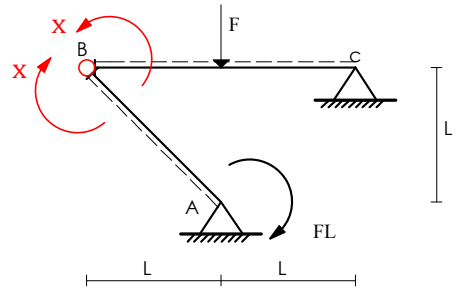


$M \Rightarrow$



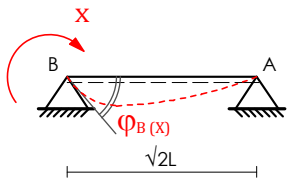
Esercizio N.25





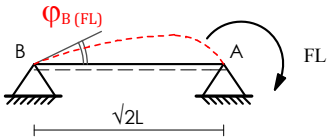
Eq. di Congruenza: $\boxed{\Delta\varphi_B = 0} \Rightarrow \boxed{\varphi_{BA} = \varphi_{BC}} \Rightarrow \varphi_{BA\,(X)} + \varphi_{BA\,(FL)} = \varphi_{BC\,(X)} + \varphi_{BC\,(F)}$

$\boxed{\varphi_{BA\,(X)}} \Rightarrow$



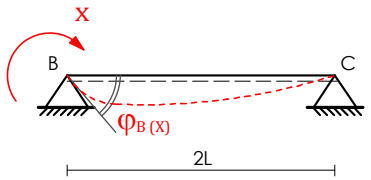
$$\boxed{\varphi_{BA\,(X)} = -\frac{XL\sqrt{2}}{3\,EJ}}$$

$\boxed{\varphi_{BA\,(FL)}} \Rightarrow$



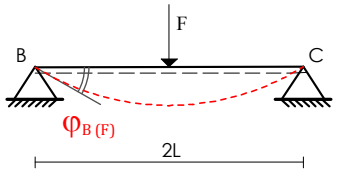
$$\boxed{\varphi_{BA\,(FL)} = \frac{FL\sqrt{2}L}{6\,EJ}}$$

$\boxed{\varphi_{BA\,(X)}} \Rightarrow$



$$\boxed{\varphi_{BA\,(X)} = \frac{X2L}{3\,EJ}}$$

$\boxed{\varphi_{BA\,(F)}} \Rightarrow$



$$\boxed{\varphi_{BA\,(X)} = -\frac{F\,(2L)^2}{16\,EJ}}$$

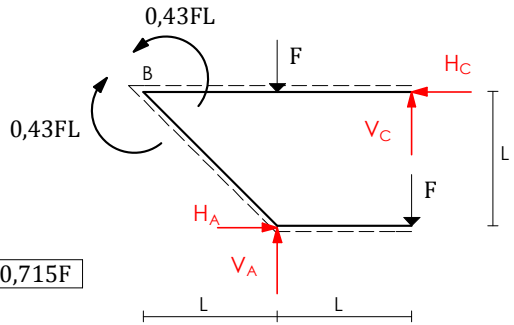
Eq. di Congruenza:

$$\varphi_{BA\,(X)} + \varphi_{BA\,(FL)} = \varphi_{BC\,(X)} + \varphi_{BC\,(F)} \quad \Rightarrow \quad -\frac{XL\sqrt{2}}{3\,EJ} + \frac{FL^2\sqrt{2}}{6\,EJ} = \frac{2XL}{3\,EJ} - \frac{FL^2}{4\,EJ}$$

$$X = \frac{5{,}828FL^2}{12\,EJ} \cdot \frac{3\,EJ}{3{,}414L} = \boxed{0{,}43FL}$$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B



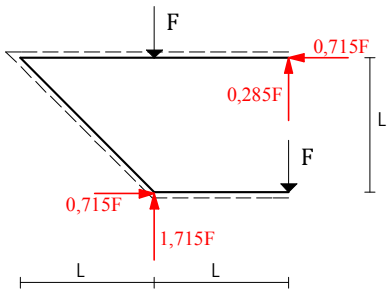
$\Sigma M_B = 0$
$0{,}43FL + V_c \cdot 2L - F \cdot L = 0$
$\rightarrow \boxed{V_C = 0{,}285F}$

$\Sigma V = 0$
$V_A + 0{,}285 - 2F = 0 \rightarrow \boxed{V_A = 1{,}715F}$

$\Sigma M_B = 0$
$-0{,}43FL + H_A \cdot L + 1{,}715F \cdot L = 0 \rightarrow \boxed{H_A = 0{,}715F}$

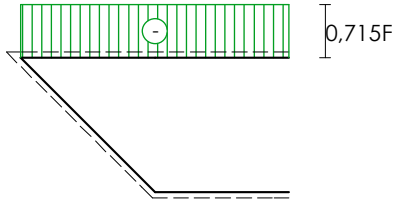
$\Sigma H = 0$
$H_C - 0{,}715F = 0 \rightarrow \boxed{H_C = 0{,}715F}$

Diagramma di corpo libero :

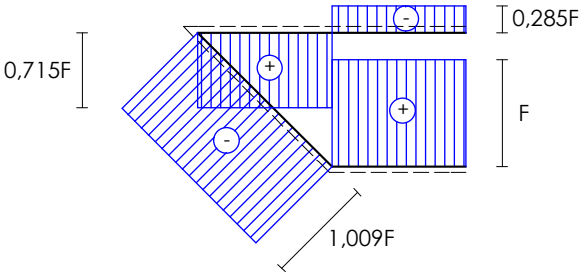


Diagrammi (N, T, M) :

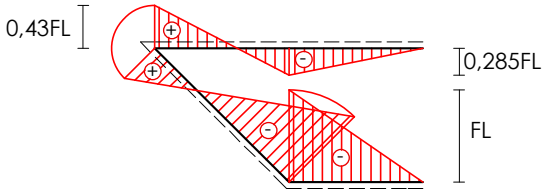
$N \Rightarrow$



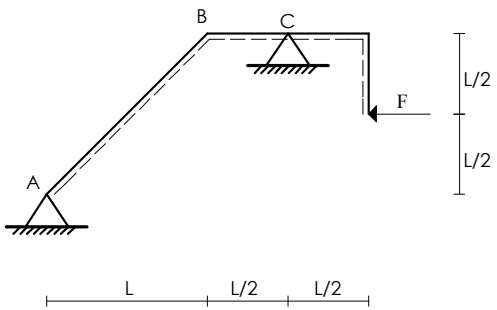
$T \Rightarrow$

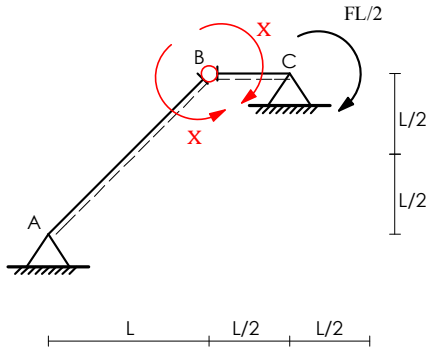


$M \Rightarrow$



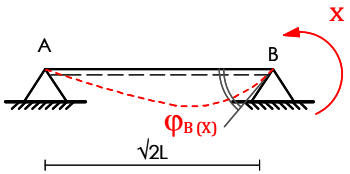
Esercizio N.26





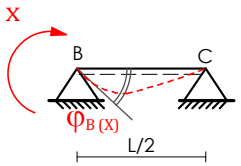
Eq. di Congruenza: $\Delta\varphi_B = 0 \Rightarrow \varphi_{BA} = \varphi_{BC} \Rightarrow \varphi_{BA\,(X)} = \varphi_{BC\,(X)} + \varphi_{BC\,(FL/2)}$

$\varphi_{BA\,(X)} \Rightarrow$



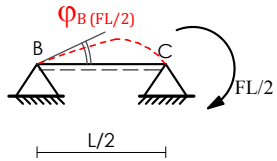
$$\varphi_{BA\,(X)} = \frac{XL\sqrt{2}}{3\,EJ}$$

$\varphi_{BC\,(X)} \Rightarrow$



$$\varphi_{BC\,(X)} = -\frac{XL/2}{3\,EJ}$$

$\varphi_{BC\,(FL/2)} \Rightarrow$



$$\varphi_{BC\,(FL/2)} = \frac{FL/2\ (L/2)}{6\,EJ}$$

Esercizio N.26

Eq. di Congruenza:

$$\varphi_{BA\,(X)} = \varphi_{BC\,(X)} + \varphi_{BC\,(FL/2)} \Rightarrow \frac{XL\sqrt{2}}{3\,EJ} = -\frac{XL}{6\,EJ} + \frac{FL^2}{24\,EJ}$$

$X = 0{,}042L^2/EJ \cdot 1{,}567\ EJ/L = \boxed{0{,}065FL}$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B

$\Sigma M_B = 0$
$-0{,}065FL + V_c \cdot L/2 - F \cdot L/2 = 0$
$\rightarrow \boxed{V_C = 1{,}131F}$

$\Sigma V = 0$
$-V_A + 1{,}131F = 0 \rightarrow \boxed{V_A = 1{,}131F}$

$\Sigma M_B = 0$
$0{,}065FL - H_A \cdot L + 1{,}131F \cdot L = 0$
$\rightarrow \boxed{H_A = 1{,}196F}$

$\Sigma H = 0$
$H_C + F - 1{,}196F = 0 \rightarrow \boxed{H_C = 2{,}196F}$

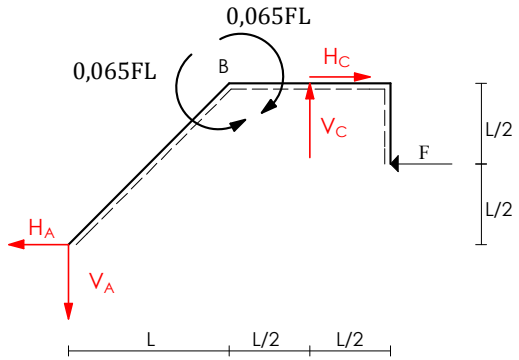
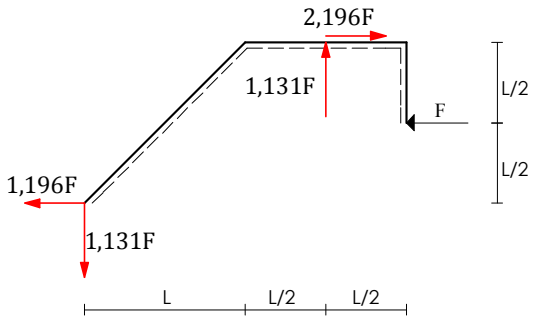


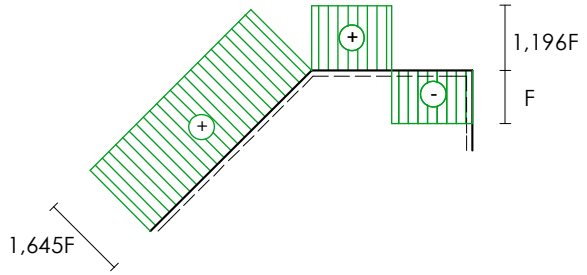
Diagramma di corpo libero :



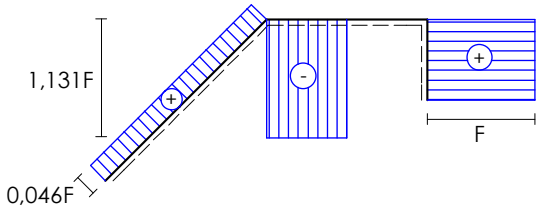
Esercizio N.26

Diagrammi (N, T, M) :

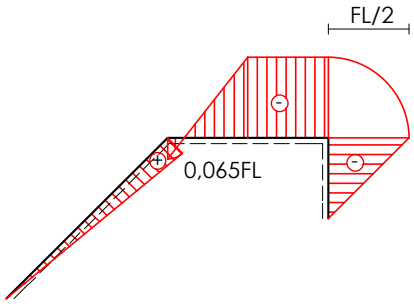
$N \Rightarrow$



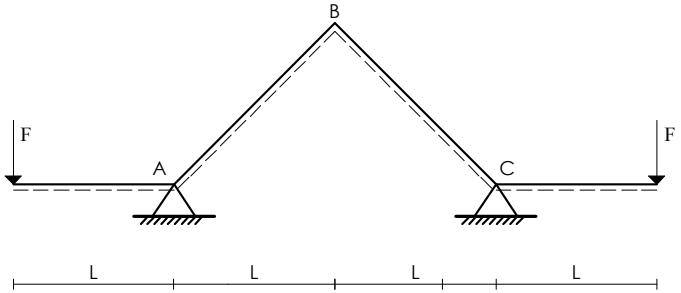
$T \Rightarrow$

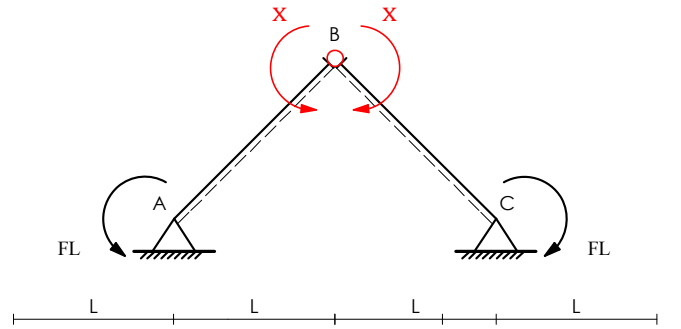


$M \Rightarrow$



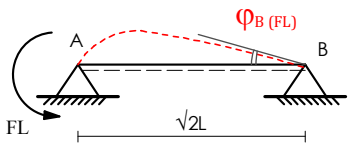
Esercizio N.27





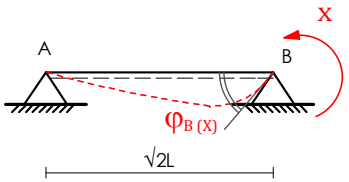
Eq. di Congruenza: $\boxed{\Delta\varphi_B = 0} \Rightarrow \boxed{\varphi_{BA} = \varphi_{BC}} \Rightarrow \varphi_{BA\,(X)} + \varphi_{BA\,(FL)} = \varphi_{BC\,(X)} + \varphi_{BC\,(FL)}$

$\boxed{\varphi_{BA\,(FL)}} \Rightarrow$



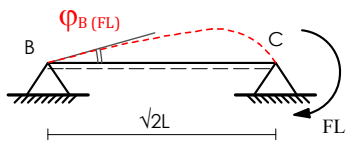
$$\varphi_{BA\,(FL)} = -\frac{FL^2\sqrt{2}}{6\,EJ}$$

$\boxed{\varphi_{BA\,(X)}} \Rightarrow$



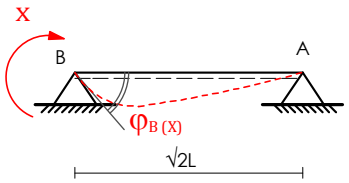
$$\varphi_{BA\,(X)} = \frac{XL\sqrt{2}}{3\,EJ}$$

$\varphi_{BC\,(FL)}$ ⇒



$$\varphi_{BC\,(FL)} = \frac{FL^2\sqrt{2}}{6\,EJ}$$

$\varphi_{BC\,(X)}$ ⇒



$$\varphi_{BC\,(X)} = -\frac{XL\sqrt{2}}{3\,EJ}$$

Eq. di Congruenza:

$$\varphi_{BA\,(X)} + \varphi_{BA\,(FL)} = \varphi_{BC\,(X)} + \varphi_{BC\,(FL)} \Rightarrow \frac{XL\sqrt{2}}{3\,EJ} - \frac{FL^2\sqrt{2}}{6\,EJ} = \frac{FL^2\sqrt{2}}{6\,EJ} - \frac{XL\sqrt{2}}{3\,EJ}$$

$$X = \frac{FL^2\sqrt{2}}{3\,EJ} \cdot \frac{3\,EJ}{2L\sqrt{2}} = \boxed{FL/2}$$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : $M_A\,M_B$

$\Sigma M_A = 0$
$FL + V_C \cdot 2L - F \cdot 3L = 0$
→ $\boxed{V_C = F}$ $\boxed{V_A = F}$

$\Sigma M_B = 0$
$-FL/2 + V_C \cdot L + H_C \cdot L - 2FL = 0$
→ $\boxed{H_C = 3/2F}$ $\boxed{H_A = 3/2F}$

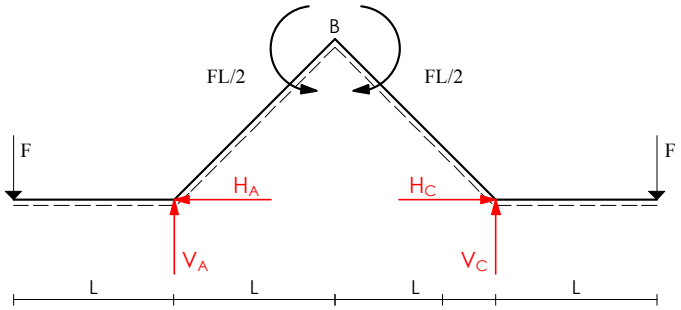
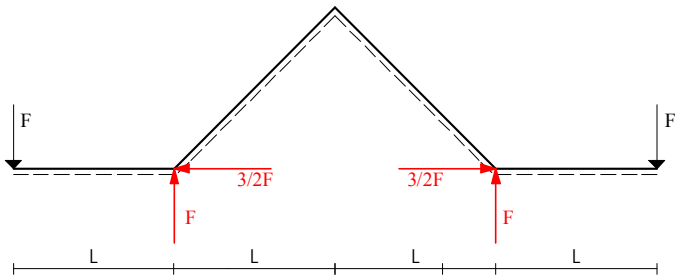
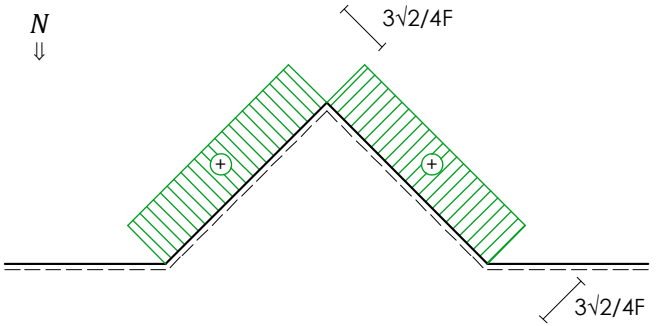


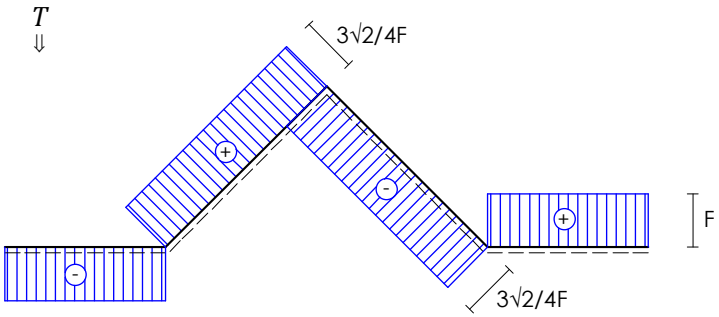
Diagramma di corpo libero :

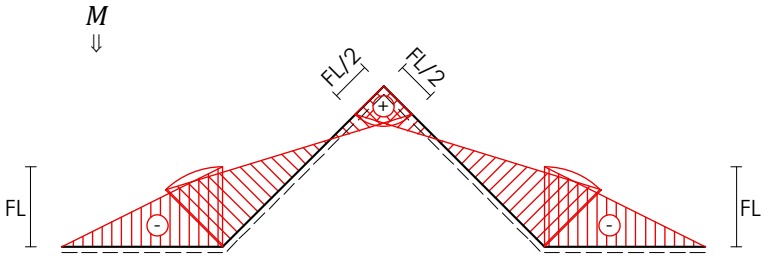


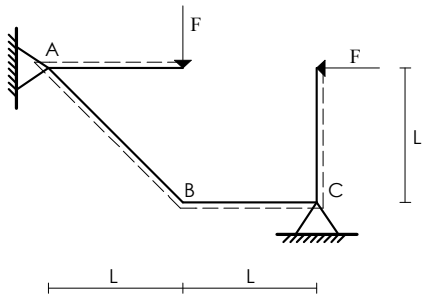
Esercizio N.27

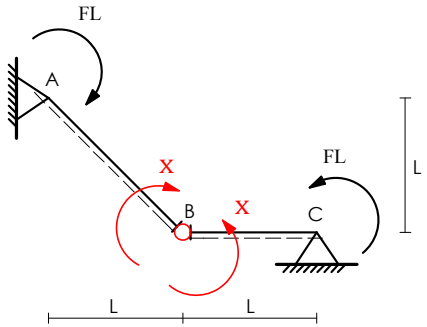
Diagrammi (N, T, M) :





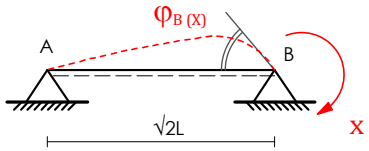






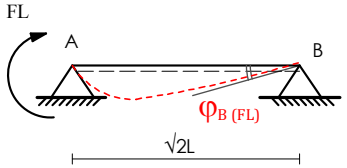
Eq. di Congruenza: $\boxed{\Delta\varphi_B = 0}$ ⇒ $\boxed{\varphi_{BA} = \varphi_{BC}}$ ⇒ $\varphi_{BA\,(X)} + \varphi_{BA\,(FL)} = \varphi_{BC\,(X)} + \varphi_{BC\,(FL)}$

$\boxed{\varphi_{BA\,(X)}}$ ⇒



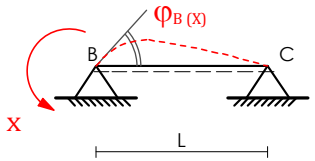
$$\boxed{\varphi_{BA\,(X)} = -\frac{XL\sqrt{2}}{3\,EJ}}$$

$\boxed{\varphi_{BA\,(FL)}}$ ⇒



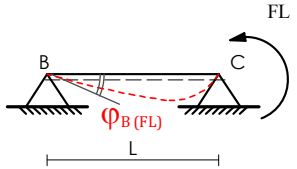
$$\boxed{\varphi_{BA\,(FL)} = \frac{FL^2\sqrt{2}}{6\,EJ}}$$

$\boxed{\varphi_{BC\,(X)}}$ ⇒



$$\boxed{\varphi_{BC\,(X)} = \frac{XL}{3\,EJ}}$$

$\boxed{\varphi_{BC\,(FL)}}$ ⇒



$$\boxed{\varphi_{BC\,(FL)} = -\frac{FL(L)}{6\,EJ}}$$

Eq. di Congruenza:

$$\varphi_{BA\,(X)} + \varphi_{BA\,(FL)} = \varphi_{BC\,(X)} + \varphi_{BC\,(FL)} \Rightarrow -\frac{XL\sqrt{2}}{3\,EJ} + \frac{FL^2\sqrt{2}}{6\,EJ} = \frac{XL}{3\,EJ} - \frac{FL^2}{6\,EJ}$$

$$X = \frac{FL^2(\sqrt{2}+1)}{6\,EJ} \cdot \frac{3\,EJ}{(\sqrt{2}+1)L} = \boxed{FL/2}$$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B

$\Sigma M_B = 0$
$FL/2 - V_c \cdot L + F \cdot L = 0 \rightarrow \boxed{V_C = 3/2F}$

$\Sigma V = 0$
$V_A - F - 3/2F = 0 \rightarrow \boxed{V_A = 5/2F}$

$\Sigma M_B = 0$
$-FL/2 + H_A \cdot L - 5/2F \cdot L = 0 \rightarrow \boxed{H_A = 3F}$

$\Sigma H = 0$
$-H_C + F + 3F = 0 \rightarrow \boxed{H_C = 4F}$

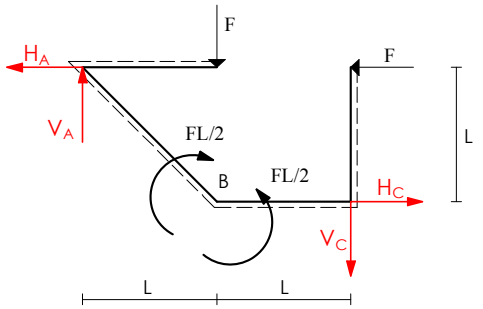
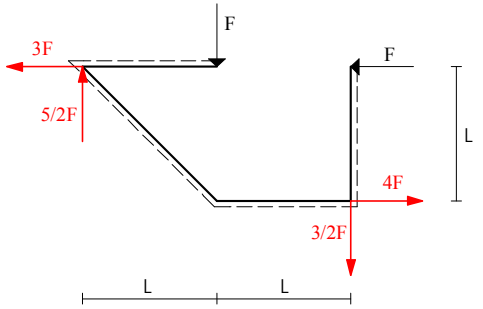


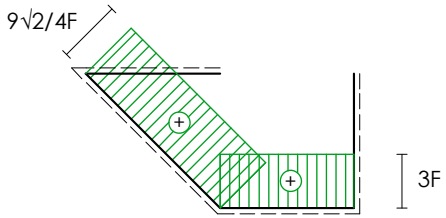
Diagramma di corpo libero :



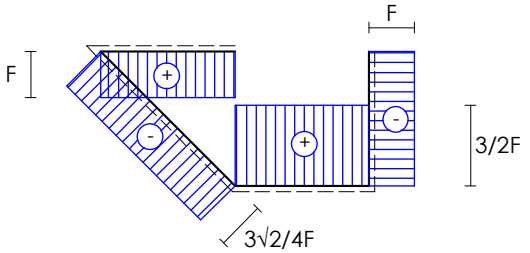
Esercizio N.28

Diagrammi (N, T, M) :

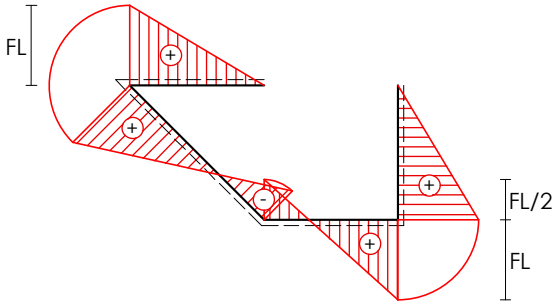
N ⇓



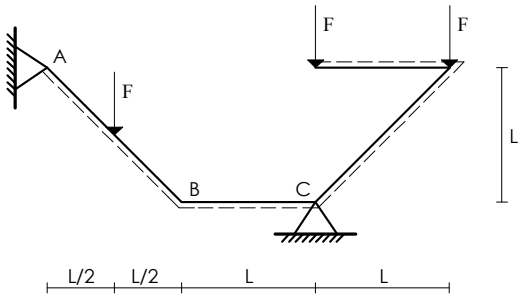
T ⇓

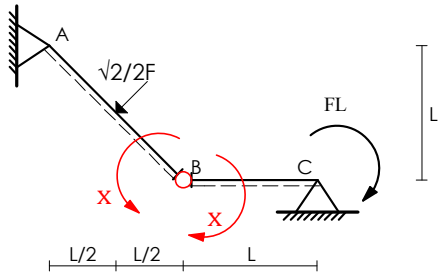


M ⇓



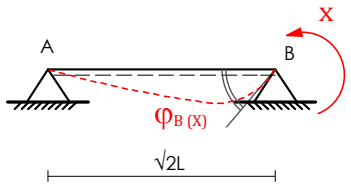
Esercizio N.29





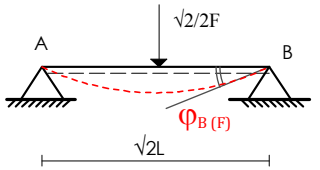
Eq. di Congruenza: $\boxed{\Delta\varphi_B = 0} \Rightarrow \boxed{\varphi_{BA} = \varphi_{BC}} \Rightarrow \varphi_{BA\,(X)} + \varphi_{BA\,(F)} = \varphi_{BC\,(X)} + \varphi_{BC\,(FL)}$

$\boxed{\varphi_{BA\,(X)}} \Rightarrow$



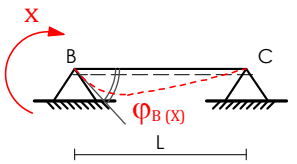
$$\varphi_{BA\,(X)} = \frac{XL\sqrt{2}}{3\,EJ}$$

$\boxed{\varphi_{BA\,(FL)}} \Rightarrow$



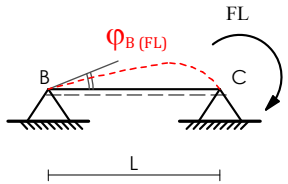
$$\varphi_{BA\,(F)} = \frac{\sqrt{2}/2F(\sqrt{2}L)^2}{16\,EJ}$$

$\boxed{\varphi_{BC\,(X)}} \Rightarrow$



$$\varphi_{BC\,(X)} = -\frac{XL}{3\,EJ}$$

$\varphi_{BC\,(X)}$ ⇒



$$\varphi_{BC\,(X)} = \frac{FL(L)}{6\,EJ}$$

Eq. di Congruenza:

$$\varphi_{BA\,(X)} + \varphi_{BA\,(F)} = \varphi_{BC\,(X)} + \varphi_{BC\,(FL)} \quad \Rightarrow \quad \frac{XL\sqrt{2}}{3\,EJ} + \frac{\sqrt{2}FL^2}{16\,EJ} = -\frac{XL}{3\,EJ} + \frac{FL^2}{6\,EJ}$$

$$X = \frac{0{,}235FL^2}{EJ} \cdot \frac{0{,}414EJ}{L} = \boxed{0{,}097FL}$$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B

$\Sigma M_B = 0$
$-0{,}097FL + V_c \cdot L - F \cdot L - F \cdot 2L = 0 \rightarrow \boxed{V_C = 3{,}097F}$

$\Sigma V = 0$
$-V_A - F - F - F - F + 3{,}097F = 0 \rightarrow \boxed{V_A = 0{,}097F}$

$\Sigma M_B = 0$
$+ 0{,}097FL - H_A \cdot L + F \cdot L + 0{,}097F \cdot L = 0$
$\rightarrow \boxed{H_A = 0{,}694F}$

$\Sigma H = 0$
$0{,}694F - H_C = 0 \rightarrow \boxed{H_C = 0{,}694F}$

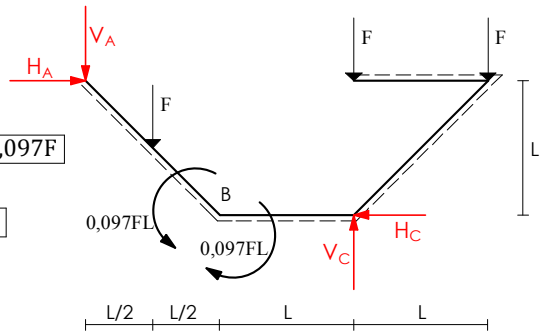
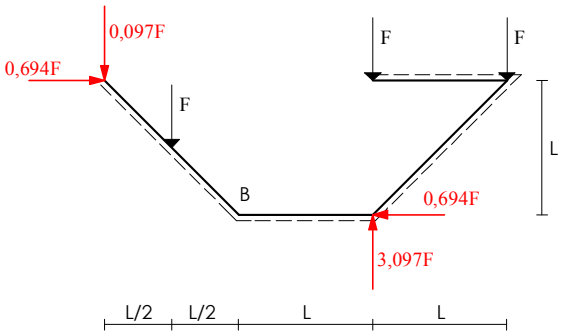
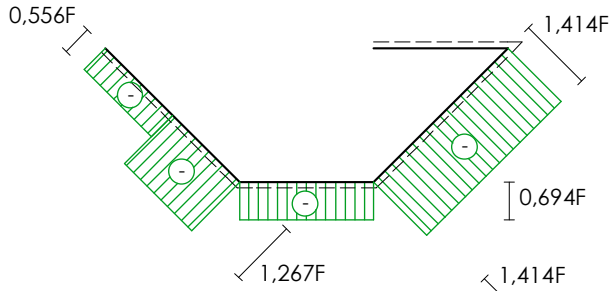


Diagramma di corpo libero :

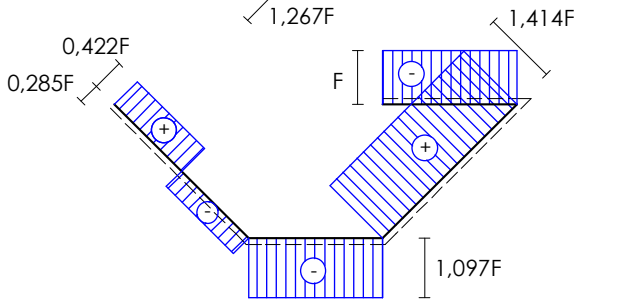


Diagrammi (N, T, M) :

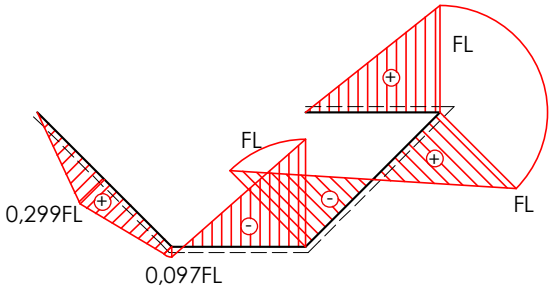
N ⇓



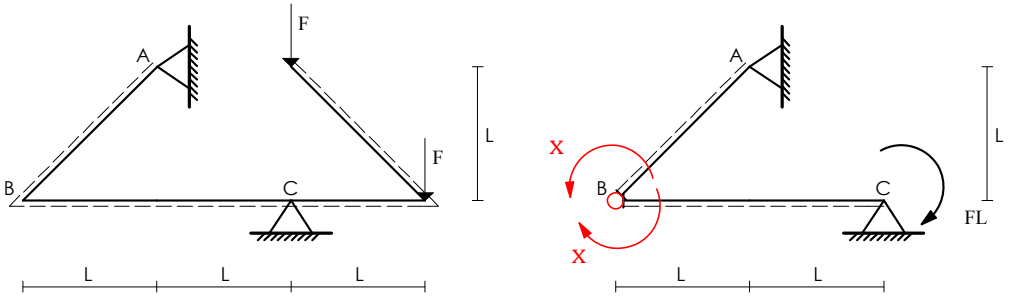
T ⇓



M ⇓

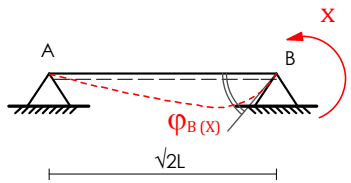


Esercizio N.30



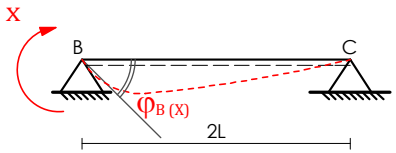
Eq. di Congruenza: $\Delta\varphi_B = 0 \Rightarrow \varphi_{BA} = \varphi_{BC} \Rightarrow \varphi_{BA\,(X)} = \varphi_{BC\,(X)} + \varphi_{BC\,(FL)}$

$\varphi_{BA\,(X)} \Rightarrow$



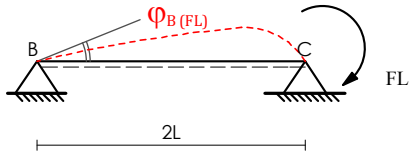
$$\varphi_{BA\,(X)} = \frac{XL\sqrt{2}}{3\,EJ}$$

$\varphi_{BC\,(X)} \Rightarrow$



$$\varphi_{BC\,(X)} = -\frac{X2L}{3EJ}$$

$\varphi_{BC\,(FL)} \Rightarrow$



$$\varphi_{BC\,(FL)} = \frac{FL(2L)}{6EJ}$$

Esercizio N.30

Eq. di Congruenza:

$$\varphi_{BA\,(X)} = \varphi_{BC\,(X)} + \varphi_{BC\,(FL)} \Rightarrow \frac{XL\sqrt{2}}{3\,EJ} = -\frac{2\,XL}{3\,EJ} + \frac{FL^2}{3\,EJ}$$

$X = 0{,}879L^2/EJ \cdot 0{,}333\,EJ/L = \boxed{0{,}293FL}$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B

$\Sigma M_B = 0$
$-0{,}293FL + V_c \cdot 2L - F \cdot 2L - F \cdot 3L = 0$
$\rightarrow \boxed{V_C = 2{,}646F}$

$\Sigma V = 0$
$-V_A + 2{,}646F - F - F = 0$
$\rightarrow \boxed{V_A = 0{,}646F}$

$\Sigma M_B = 0$
$0{,}293FL + H_A \cdot L - 0{,}646F \cdot L = 0$
$\rightarrow \boxed{H_A = 0{,}354F}$

$\Sigma H = 0$
$-H_C + 0{,}354F = 0 \rightarrow \boxed{H_C = 0{,}354F}$

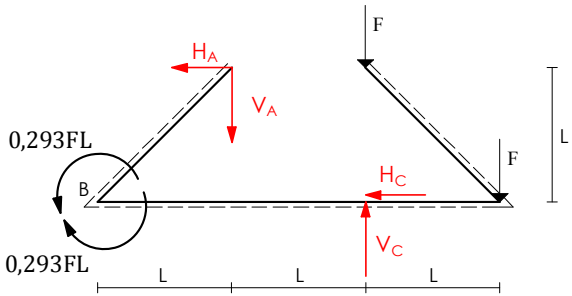
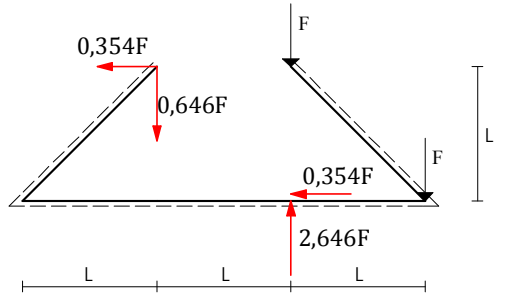


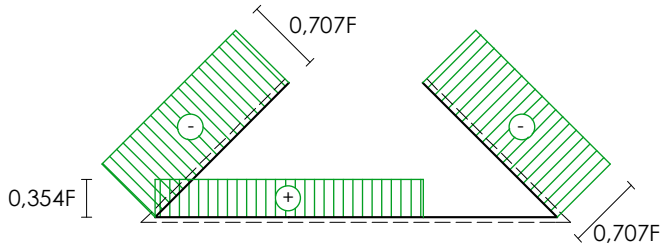
Diagramma di corpo libero :



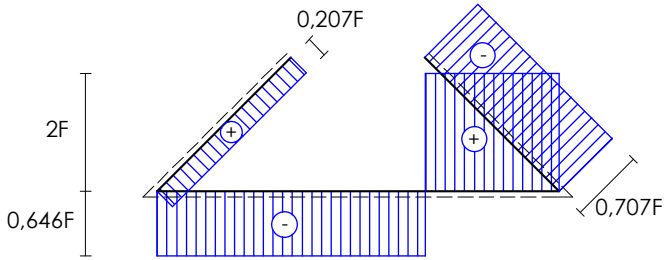
Esercizio N.30

Diagrammi (N, T, M) :

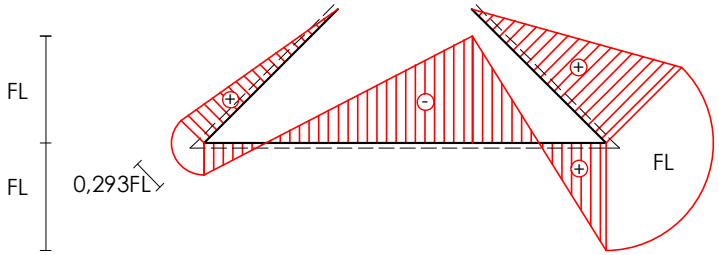
$N \Rightarrow$



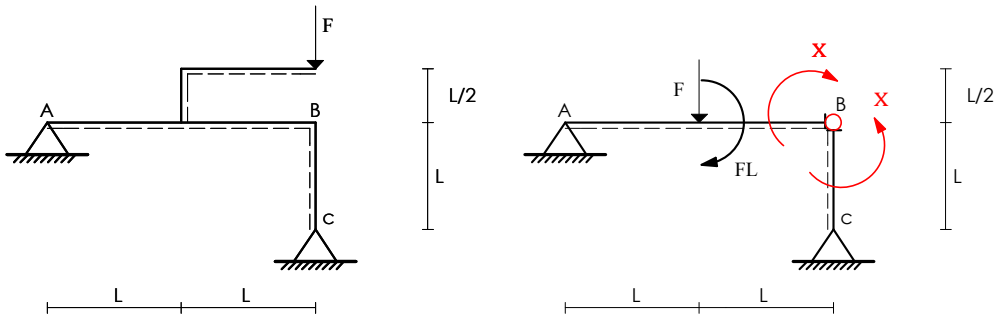
$T \Rightarrow$



$M \Rightarrow$

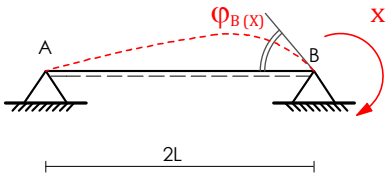


Esercizio N.31



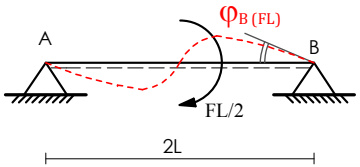
Eq. di Congruenza: $\boxed{\Delta\varphi_B = 0}$ ⇒ $\boxed{\varphi_{BA} = \varphi_{BC}}$ ⇒ $\varphi_{BA\,(X)} + \varphi_{BA\,(FL)} + \varphi_{BA\,(F)} = \varphi_{BC\,(X)}$

$\boxed{\varphi_{BA\,(X)}}$ ⇒



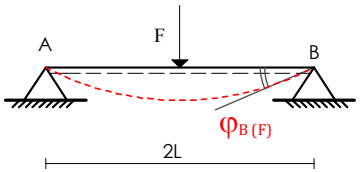
$$\varphi_{BA\,(X)} = -\frac{X2L}{3\,EJ}$$

$\boxed{\varphi_{BA\,(FL)}}$ ⇒



$$\varphi_{BA\,(FL)} = -\frac{FL(2L)}{24EJ}$$

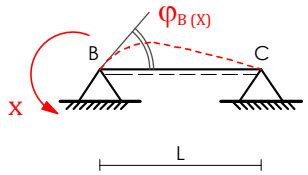
$\boxed{\varphi_{BA\,(F)}}$ ⇒



$$\varphi_{BA\,(F)} = \frac{F(2L)^2}{16EJ}$$

Esercizio N.31

$\boxed{\varphi_{BC\,(X)}}$ ⇒



$$\varphi_{BC\,(X)} = \frac{XL}{3\,EJ}$$

Eq. di Congruenza:

$$\varphi_{BA\,(X)} + \varphi_{BA\,(FL)} + \varphi_{BA\,(F)} = \varphi_{BC\,(X)} \Rightarrow -\frac{X2L}{3\,EJ} - \frac{FL^2}{12\,EJ} + \frac{FL^2}{4\,EJ} = \frac{XL}{3\,EJ}$$

$$X = \frac{FL^2}{3\,EJ} \cdot \frac{EJ}{L} = \boxed{FL/3}$$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B

$\Sigma M_B = 0$
$-FL/3 + V_A \cdot 2L = 0 \rightarrow \boxed{V_A = F/6}$

$\Sigma V = 0$
$F/6 + F - V_C = 0 \rightarrow \boxed{V_C = 7/6F}$

$\Sigma M_B = 0$
$FL/3 + H_C \cdot L = 0 \rightarrow \boxed{H_C = F/3}$ $\boxed{H_A = F/3}$

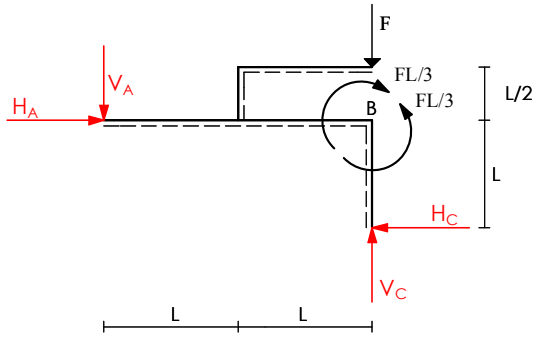
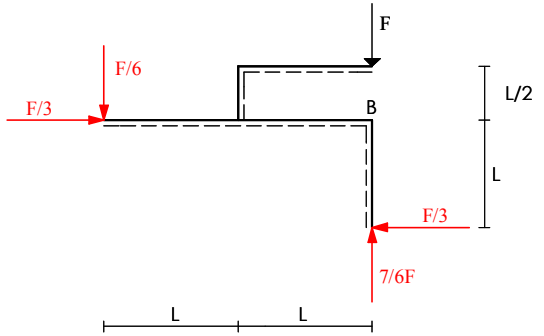


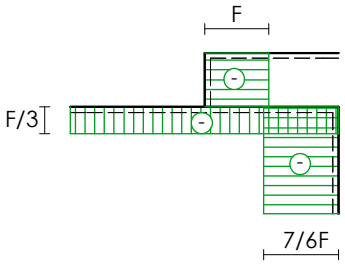
Diagramma di corpo libero :



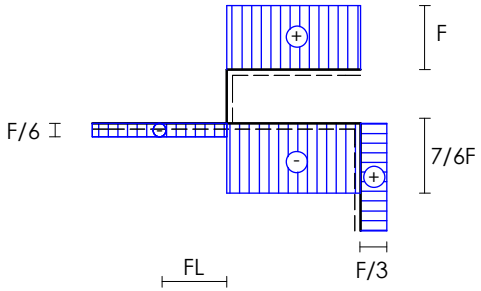
Esercizio N.31

Diagrammi (N, T, M) :

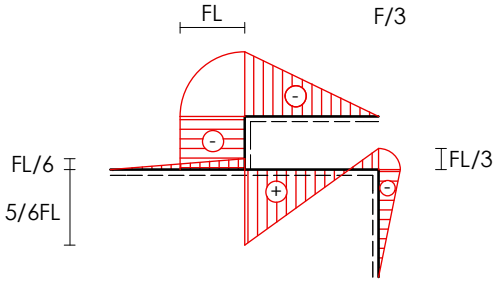
N ⇓



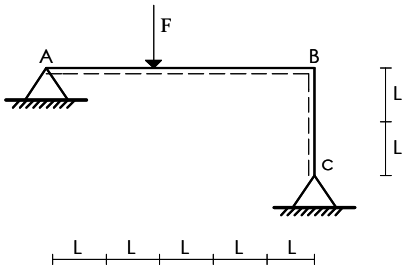
T ⇓

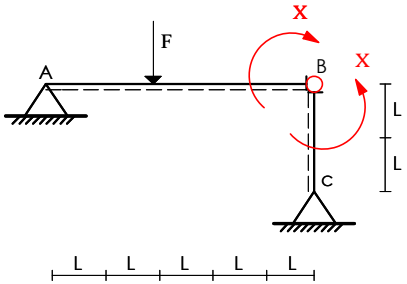


M ⇓



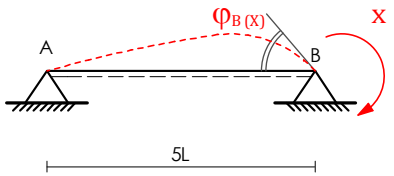
Esercizio N.32





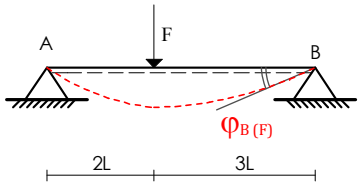
Eq. di Congruenza: $\boxed{\Delta\varphi_B = 0}$ ⇒ $\boxed{\varphi_{BA} = \varphi_{BC}}$ ⇒ $\varphi_{BA\,(X)} + \varphi_{BA\,(F)} = \varphi_{BC\,(X)}$

$\boxed{\varphi_{BA\,(X)}}$ ⇒



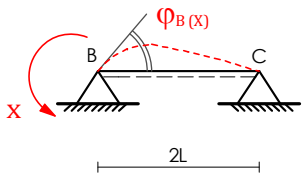
$$\varphi_{BA\,(X)} = -\frac{X5L}{3\,EJ}$$

$\boxed{\varphi_{BA\,(F)}}$ ⇒



$$\varphi_{BA\,(F)} = \frac{F2L[(5L)^2-(2L)^2]}{6EJ\,(5L)}$$

$\boxed{\varphi_{BC\,(X)}}$ ⇒



$$\varphi_{BC\,(X)} = \frac{X2L}{3\,EJ}$$

Eq. di Congruenza:

$$\varphi_{BA\,(X)} + \varphi_{BA\,(F)} = \varphi_{BC\,(X)} \quad \Rightarrow \quad -\frac{X5L}{3\,EJ} + \frac{7FL^2}{5\,EJ} = \frac{X2L}{3\,EJ}$$

$$X = \frac{7FL^2}{25\,EJ} \cdot \frac{3\,EJ}{7\,L} = \boxed{3/5FL}$$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B

$\Sigma M_B = 0$

$3/5FL - H_B \cdot 2L = 0 \rightarrow \boxed{H_B = 3/10F}$ $\boxed{H_A = 3/10F}$

$\Sigma M_B = 0$

$-3/5FL - V_A \cdot 5L + F \cdot 3L = 0 \rightarrow \boxed{V_A = 12/25F}$

$\Sigma V = 0$

$12/25F - F + V_C = 0 \rightarrow \boxed{V_C = 13/25F}$

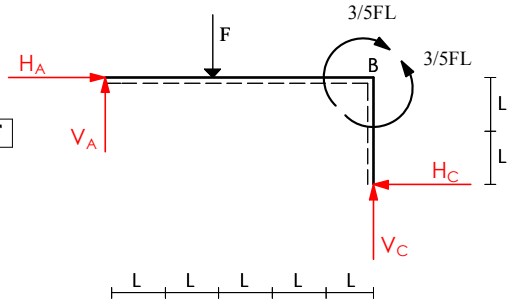
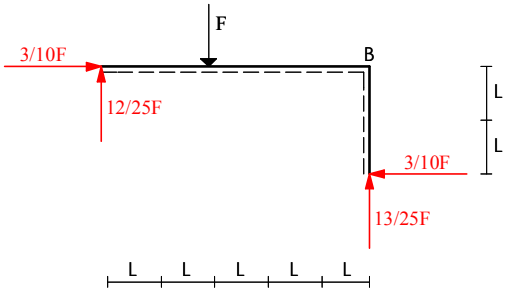
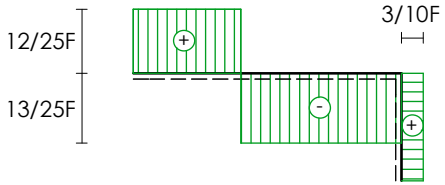


Diagramma di corpo libero :



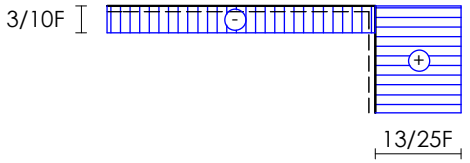
Diagrammi (N, T, M) :

N ⇓

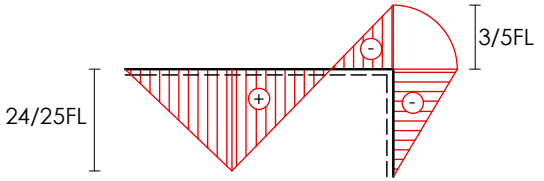


Esercizio N.32

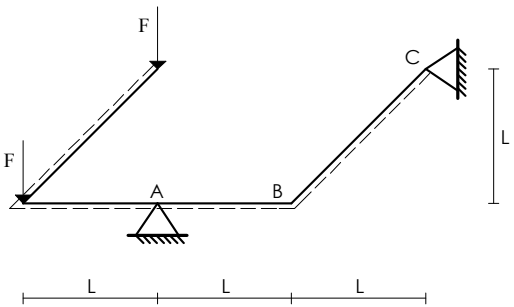
T ⇓

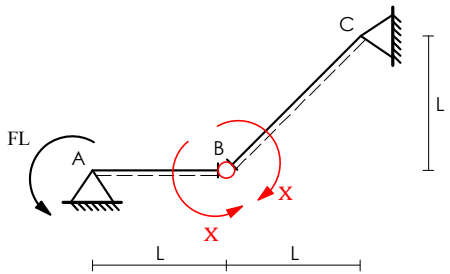


M ⇓



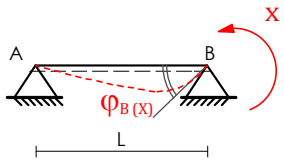
Esercizio N.33





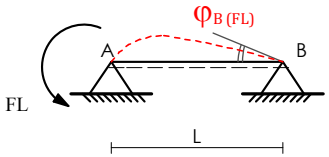
Eq. di Congruenza: $\boxed{\Delta\varphi_B = 0} \Rightarrow \boxed{\varphi_{BA} = \varphi_{BC}} \Rightarrow \varphi_{BA\,(X)} + \varphi_{BA\,(FL)} = \varphi_{BC\,(X)}$

$\boxed{\varphi_{BA\,(X)}} \Rightarrow$



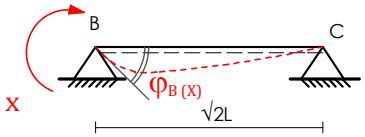
$$\varphi_{BA\,(X)} = \frac{XL}{3\,EJ}$$

$\boxed{\varphi_{BA\,(FL)}} \Rightarrow$



$$\varphi_{BA\,(FL)} = -\frac{FL(L)}{6\,EJ}$$

$\boxed{\varphi_{BC\,(X)}} \Rightarrow$



$$\varphi_{BC\,(X)} = -\frac{XL\sqrt{2}}{3\,EJ}$$

Esercizio N.33

Eq. di Congruenza:

$$\varphi_{BA\,(X)} + \varphi_{BA\,(FL)} = \varphi_{BC\,(X)} \Rightarrow \frac{XL}{3\,EJ} - \frac{FL^2}{6\,EJ} = -\frac{XL\sqrt{2}}{3\,EJ}$$

$X = 0{,}167L^2/EJ \cdot 1{,}243\,EJ/L = \boxed{0{,}207FL}$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B

$\Sigma M_B = 0$
$0{,}207FL - V_A \cdot L + F \cdot 2L + F \cdot L = 0$
$\rightarrow \boxed{V_A = 3{,}207F}$

$\Sigma V = 0$
$-V_C + 3{,}207F - F - F = 0$
$\rightarrow \boxed{V_C = 1{,}207F}$

$\Sigma M_B = 0$
$-0{,}207FL + H_C \cdot L - 1{,}207F \cdot L = 0$
$\rightarrow \boxed{H_A = 1{,}414F}$

$\Sigma H = 0$
$-H_C + 1{,}414F = 0 \rightarrow \boxed{H_C = 1{,}414F}$

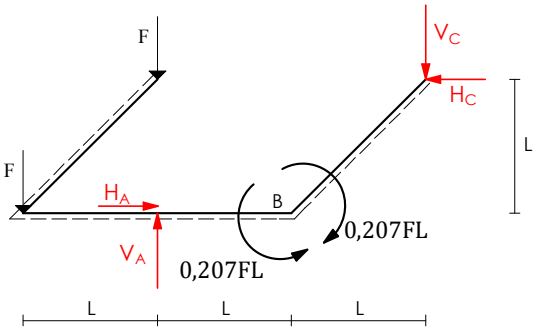
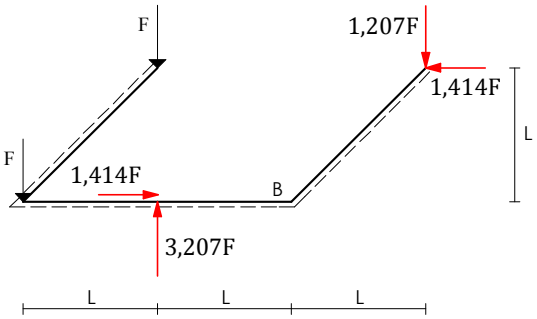


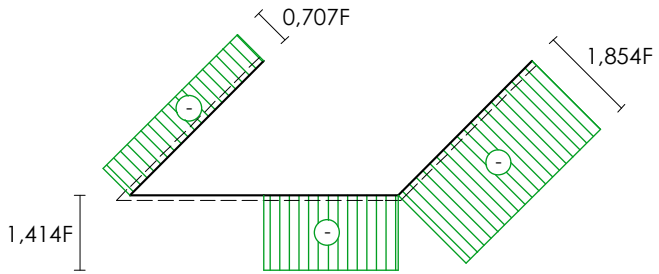
Diagramma di corpo libero :



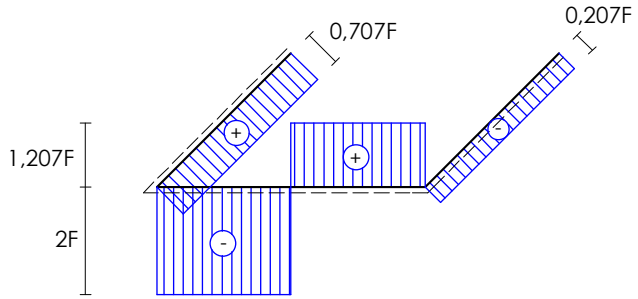
Esercizio N.33

Diagrammi (N, T, M) :

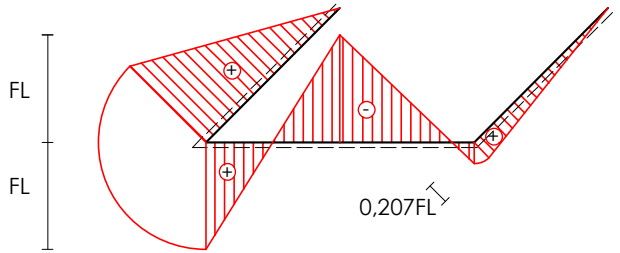
$N \Rightarrow$



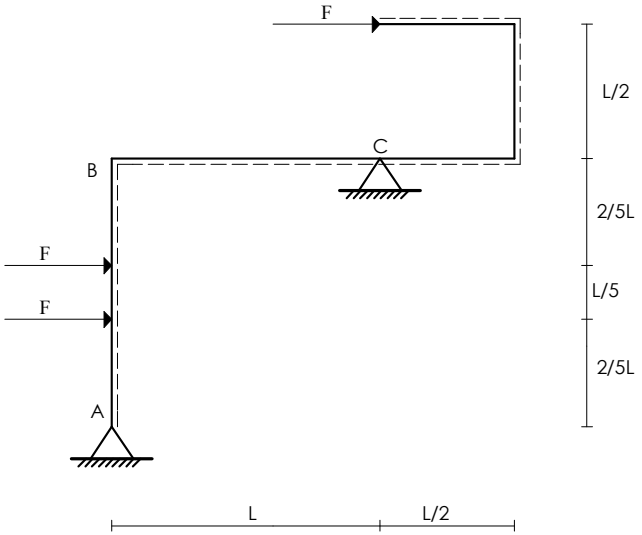
$T \Rightarrow$

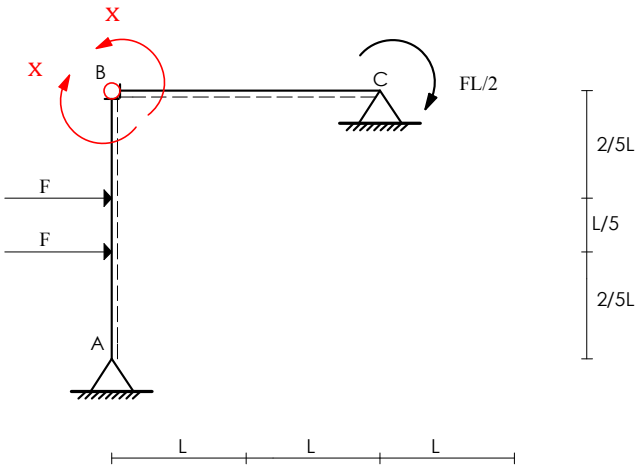


$M \Rightarrow$



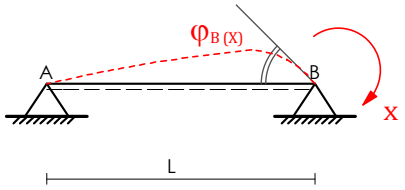
Esercizio N.34





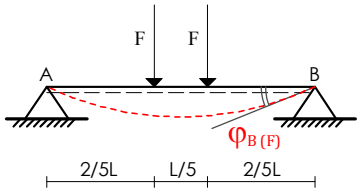
Eq. di Congruenza: $\boxed{\Delta\varphi_B = 0} \Rightarrow \boxed{\varphi_{BA} = \varphi_{BC}} \Rightarrow \varphi_{BA\,(X)} + \varphi_{BA\,(F)} = \varphi_{BC\,(X)} + \varphi_{BC\,(FL/2)}$

$\varphi_{BA\,(X)}$ ⇒



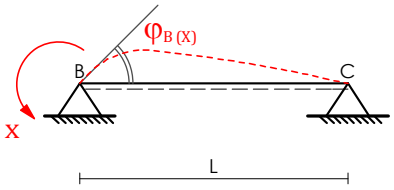
$$\varphi_{BA\,(X)} = -\frac{XL}{3EJ}$$

$\varphi_{BA\,(F)}$ ⇒



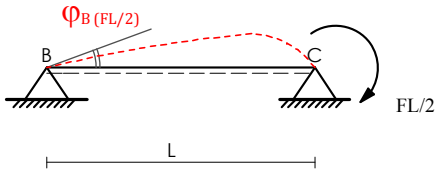
$$\varphi_{BA\,(F)} = \frac{3FL^2}{25EJ}$$

$\varphi_{BC\,(X)}$ ⇒



$$\varphi_{BC\,(X)} = \frac{XL}{3EJ}$$

$\varphi_{BC\,(FL/2)}$ ⇒



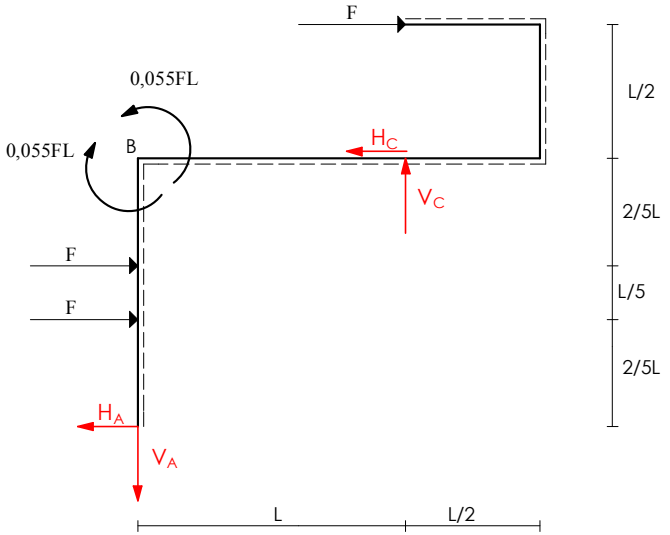
$$\varphi_{BC\,(FL/2)} = \frac{FL/2(L)}{6EJ}$$

Eq. di Congruenza:

$$\varphi_{BA\,(X)} + \varphi_{BA\,(F)} = \varphi_{BC\,(X)} + \varphi_{BC\,(FL/2)} \Rightarrow -\frac{XL}{3\,EJ} + \frac{3FL^2}{25EJ} = \frac{XL}{3\,EJ} + \frac{FL^2}{12\,EJ}$$

$$X = \frac{11FL^2}{300\,EJ} \cdot \frac{3\,EJ}{2L} = \boxed{0{,}055FL}$$

Calcolo reazioni vincolari incognite:



Eq. Ausiliaria : M_B

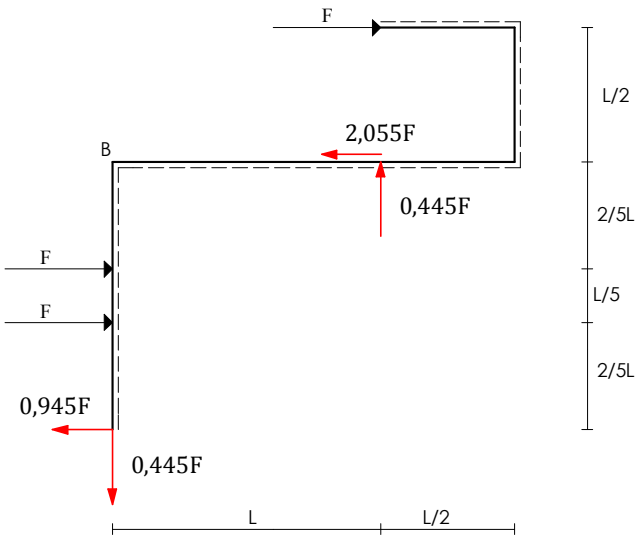
$\Sigma M_B = 0$
$0{,}055FL + V_C \cdot L - F \cdot L/2 = 0 \rightarrow \boxed{V_C = 0{,}445F}$

$\Sigma V = 0$
$-V_A + 0{,}445F = 0 \rightarrow \boxed{V_A = 0{,}445F}$

$\Sigma M_B = 0$
$-0{,}055FL - H_A \cdot L + F \cdot L/2 + F \cdot 2/5L = 0$
$\rightarrow \boxed{H_A = 0{,}945F}$

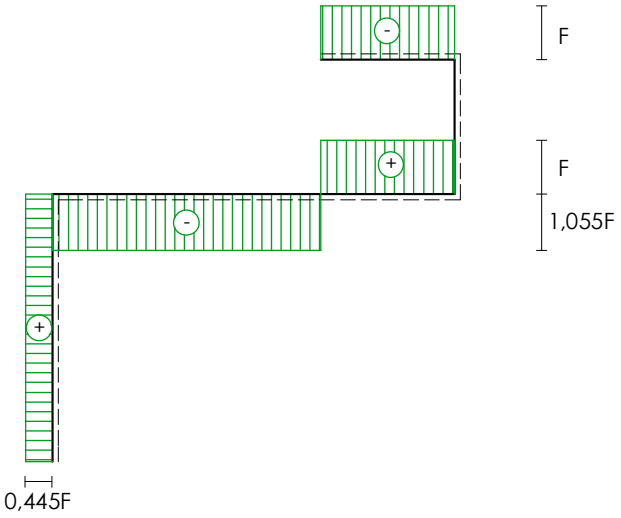
$\Sigma H = 0$
$-H_C - 0{,}945F + F + F + F = 0 \rightarrow \boxed{H_C = 2{,}055F}$

Diagramma di corpo libero :



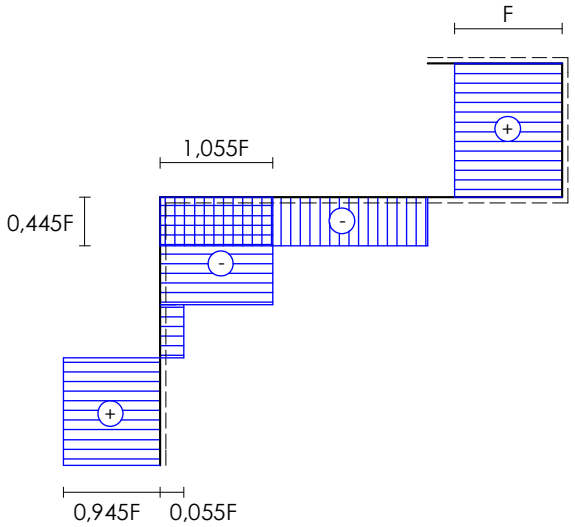
Diagrammi (N, T, M) :

$N \Rightarrow$

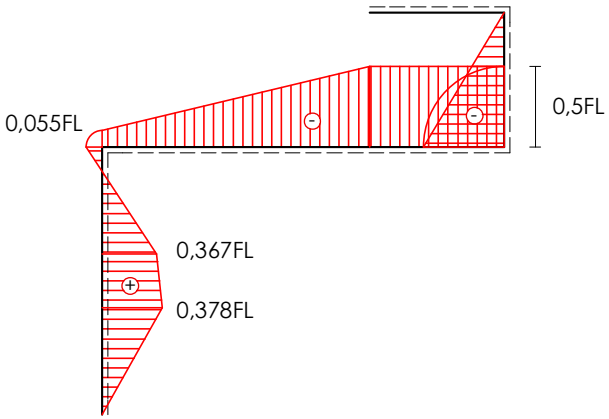


Esercizio N.34

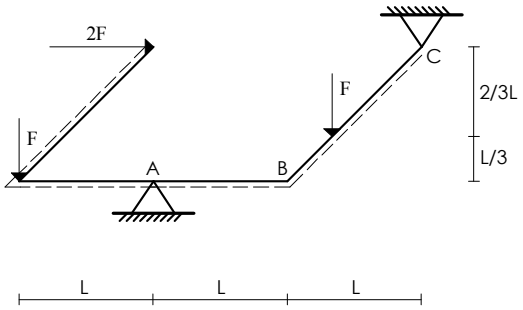
$T \Rightarrow$

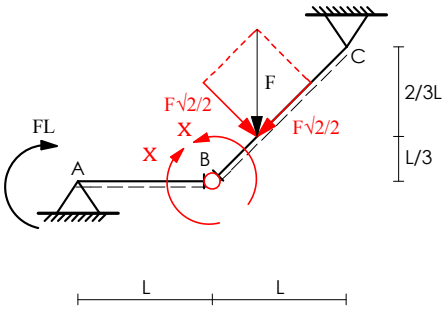


$M \Rightarrow$



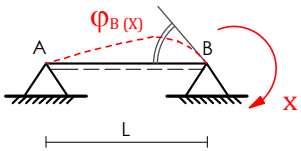
Esercizio N.35





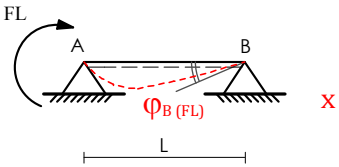
Eq. di Congruenza: $\boxed{\Delta\varphi_B = 0}$ ⇒ $\boxed{\varphi_{BA} = \varphi_{BC}}$ ⇒ $\varphi_{BA\,(X)} + \varphi_{BA\,(FL)} = \varphi_{BC\,(\sqrt{2}/2F)} + \varphi_{BC\,(X)}$

$\boxed{\varphi_{BA\,(X)}}$ ⇒



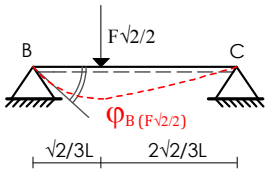
$$\varphi_{BA\,(X)} = -\frac{XL}{3\,EJ}$$

$\boxed{\varphi_{BA\,(FL)}}$ ⇒



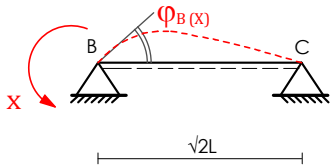
$$\varphi_{BA\,(FL)} = \frac{FL(L)}{6\,EJ}$$

$\boxed{\varphi_{BC\,(F\sqrt{2}/2)}}$ ⇒



$$\varphi_{BC\,(F\sqrt{2}/2)} = -\frac{5(F\sqrt{2}/2)(\sqrt{2}L)^2}{81\,EJ}$$

$\boxed{\varphi_{BC\,(X)}}$ ⇒



$$\varphi_{BC\,(X)} = \frac{X\sqrt{2}L}{3\,EJ}$$

Esercizio N.35

Eq. di Congruenza:

$$\varphi_{BA\,(X)} + \varphi_{BA\,(FL)} = \varphi_{BC\,(\sqrt{2}/2F)} + \varphi_{BC\,(X)} \Rightarrow -\frac{XL}{3\,EJ} + \frac{FL^2}{6\,EJ} = -\frac{5\sqrt{2}}{81\,EJ} + \frac{XL\sqrt{2}}{3\,EJ}$$

$X = 0{,}254FL \cdot 1{,}243 = \boxed{0{,}315FL}$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B

$\Sigma M_B = 0$
$-0{,}315FL + V_A \cdot L + F \cdot 2L - 2F \cdot L = 0$
$\rightarrow \boxed{V_A = 0{,}316F}$

$\Sigma V = 0$
$V_C - 0{,}316F - F = 0$
$\rightarrow \boxed{V_C = 2{,}316F}$

$\Sigma M_B = 0$
$0{,}315FL - H_C \cdot L + 2{,}316F \cdot L - F \cdot L/3 = 0$
$\rightarrow \boxed{H_C = 2{,}298F}$

$\Sigma H = 0$
$-H_A + 2{,}298F + 2F = 0 \rightarrow \boxed{H_A = 4{,}298F}$

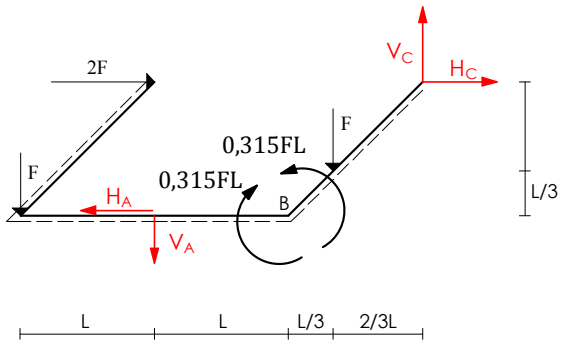
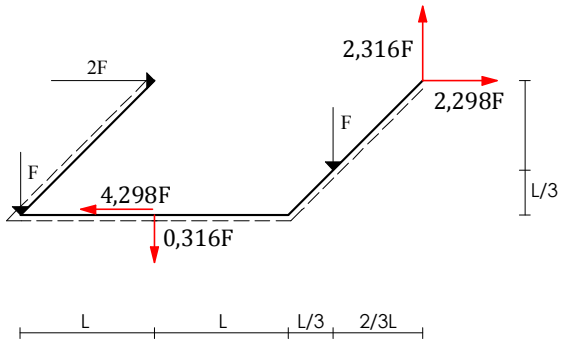


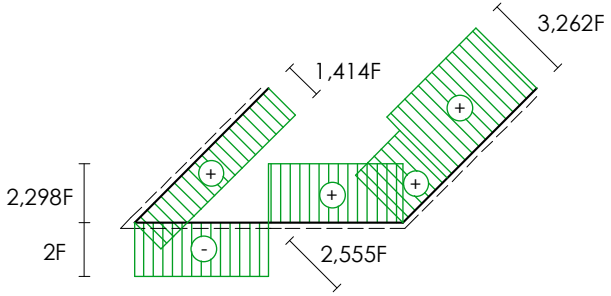
Diagramma di corpo libero :

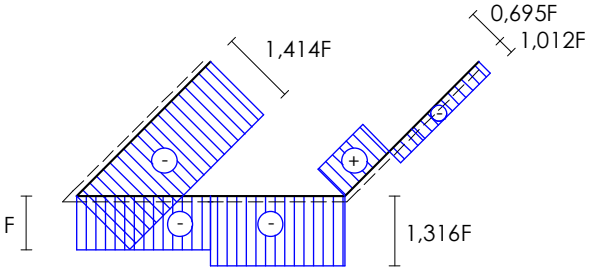


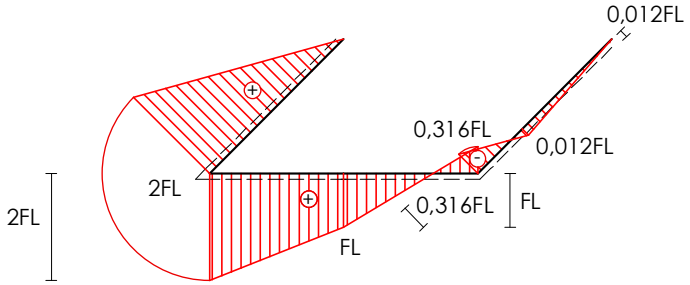
Esercizio N.35

Diagrammi (N, T, M) :

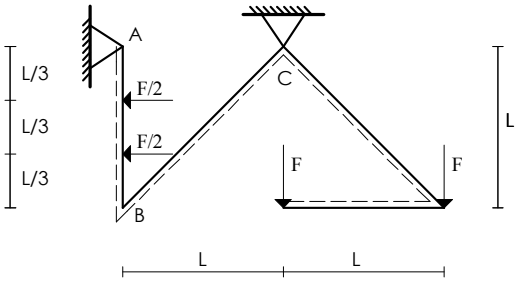
$N \Rightarrow$

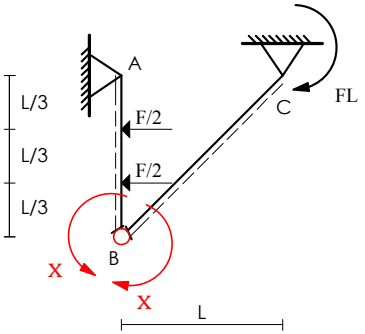






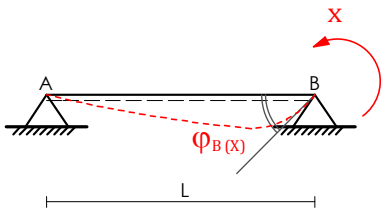
Esercizio N.36





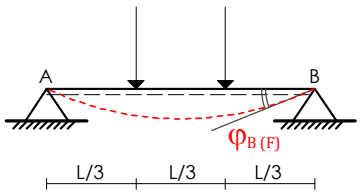
Eq. di Congruenza: $\boxed{\Delta\varphi_B = 0} \Rightarrow \boxed{\varphi_{BA} = \varphi_{BC}} \Rightarrow \varphi_{BA\,(X)} + \varphi_{BA\,(F/2)} = \varphi_{BC\,(X)} + \varphi_{BC\,(FL)}$

$\boxed{\varphi_{BA\,(X)}} \Rightarrow$



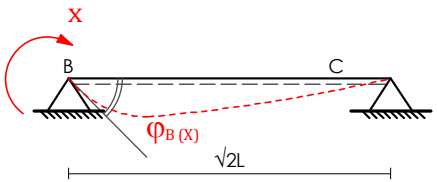
$$\varphi_{BA\,(X)} = \frac{XL}{3EJ}$$

$\boxed{\varphi_{BA\,(F/2)}} \Rightarrow$



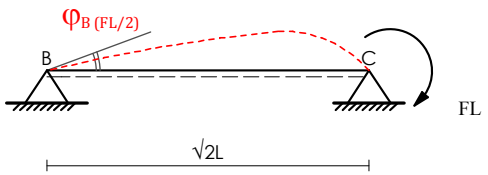
$$\varphi_{BA\,(L/2)} = \frac{F/2\,(L^2)}{9\,EJ}$$

$\boxed{\varphi_{BC\,(X)}} \Rightarrow$



$$\varphi_{BC\,(X)} = -\frac{XL\sqrt{2}}{3EJ}$$

$\varphi_{BC\,(FL/2)}$ ⇒



$$\varphi_{BC\,(FL/2)} = \frac{FL(\sqrt{2}L)}{3\,EJ}$$

Eq. di Congruenza:

$$\varphi_{BA\,(X)} + \varphi_{BA\,(F/2)} = \varphi_{BC\,(X)} + \varphi_{BC\,(FL)} \Rightarrow \frac{XL}{3\,EJ} + \frac{FL^2}{18\,EJ} = -\frac{XL\sqrt{2}}{3\,EJ} + \frac{\sqrt{2}\,FL^2}{3\,EJ}$$

$X = 0{,}541FL \cdot 0{,}414 = \boxed{0{,}224FL}$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B

$\Sigma M_B = 0$
$0{,}224FL - H_A \cdot L + F/2 \cdot 2/3L + F/2 \cdot L/3 = 0$
$\rightarrow \boxed{H_A = 0{,}724F}$

$\Sigma H = 0$
$-H_C + 0{,}724F - F/2 - F/2 = 0$
$\rightarrow \boxed{H_C = 0{,}276F}$

$\Sigma M_B = 0$
$-0{,}224FL + 0{,}276F \cdot L + V_C \cdot L - F \cdot L - F \cdot 2L = 0$
$\rightarrow \boxed{V_C = 3{,}499F}$

$\Sigma V = 0$
$-V_A + 3{,}499F - F - F = 0 \rightarrow \boxed{V_A = 1{,}499F}$

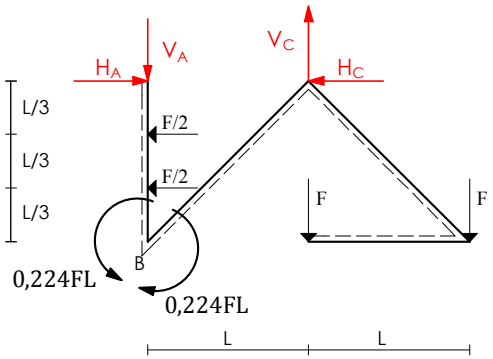
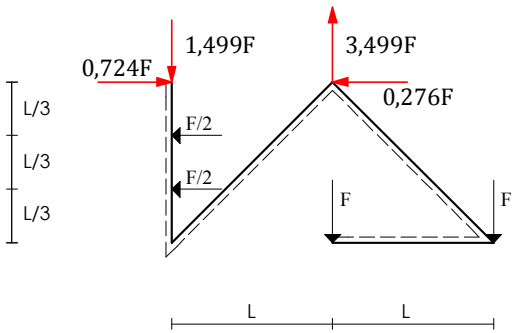
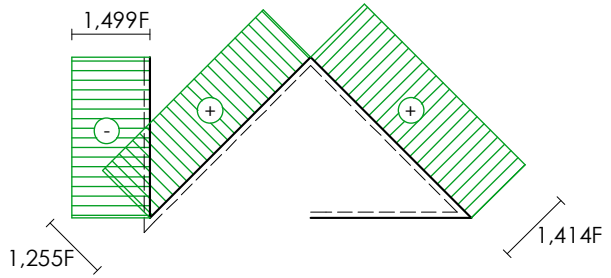


Diagramma di corpo libero :

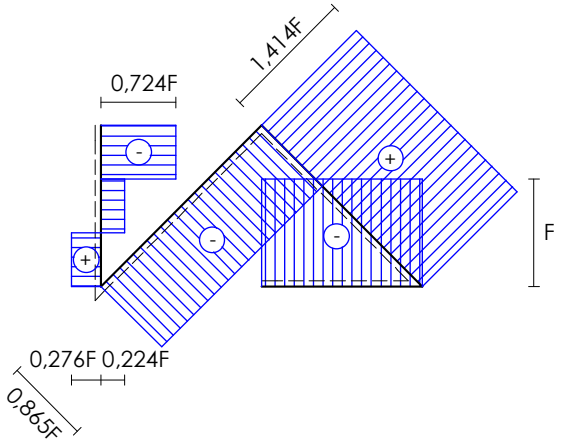


Diagrammi (N, T, M) :

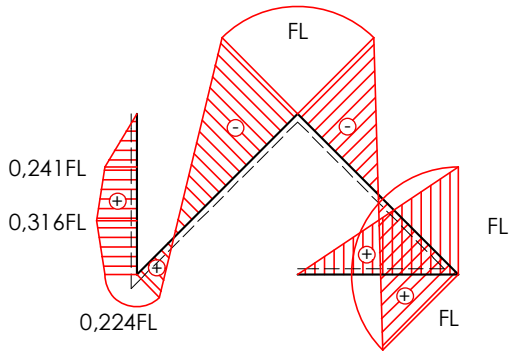
$N \Rightarrow$



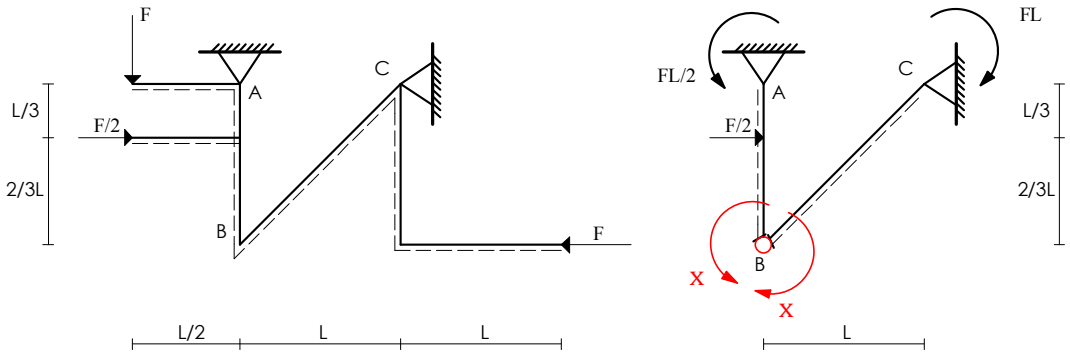
$T \Rightarrow$



$M \Rightarrow$

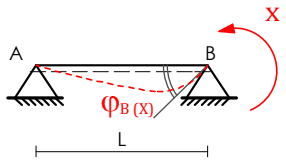


Esercizio N.37



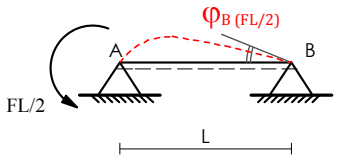
Eq. di Congruenza: $\boxed{\Delta\varphi_B = 0}$ ⇒ $\boxed{\varphi_{BA} = \varphi_{BC}}$ ⇒ $\varphi_{BA\,(X)} + \varphi_{BA\,(FL/2)} + \varphi_{BA(F/2)} = \varphi_{BC\,(X)} + \varphi_{BC\,(FL)}$

$\boxed{\varphi_{BA\,(X)}}$ ⇒



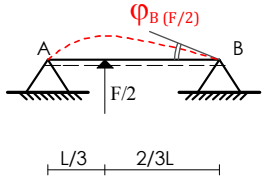
$$\boxed{\varphi_{BA\,(X)} = \frac{XL}{3\,EJ}}$$

$\boxed{\varphi_{BA\,(FL/2)}}$ ⇒



$$\boxed{\varphi_{BA\,(FL/2)} = -\frac{FL/2(L)}{6\,EJ}}$$

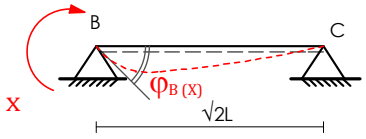
$\boxed{\varphi_{BA\,(F/2)}}$ ⇒



$$\boxed{\varphi_{BA\,(F/2)} = -\frac{4(F/2)L^2}{81\,EJ}}$$

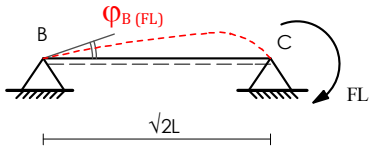
Esercizio N.37

$\boxed{\varphi_{BC\,(X)}}$ ⇒



$$\boxed{\varphi_{BC\,(X)} = -\frac{XL\sqrt{2}}{3\,EJ}}$$

$\boxed{\varphi_{BC\,(FL)}}$ ⇒



$$\boxed{\varphi_{BC\,(FL)} = \frac{FL\sqrt{2}L}{6\,EJ}}$$

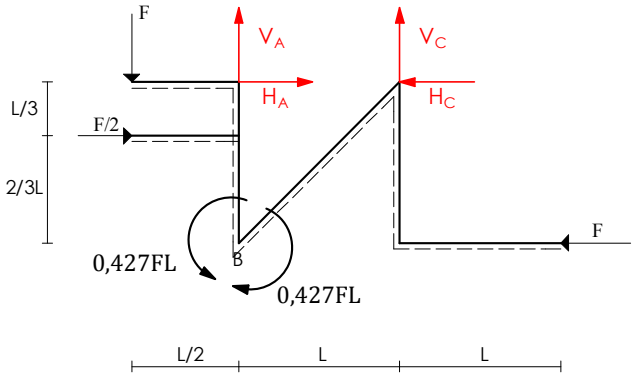
Eq. di Congruenza:

$$\varphi_{BA\,(X)} + \varphi_{BA\,(FL/2)} + \varphi_{BA(F/2)} = \varphi_{BC\,(X)} + \varphi_{BC\,(FL)} \Rightarrow$$

$$\frac{XL}{3\,EJ} - \frac{FL^2}{12\,EJ} - \frac{2FL^2}{81\,EJ} = -\frac{XL\sqrt{2}}{3\,EJ} + \frac{\sqrt{2}\,FL^2}{6\,EJ}$$

$$X = 1{,}031FL \cdot 0{,}414 = \boxed{0{,}427FL}$$

Calcolo reazioni vincolari incognite:



Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B

$\Sigma M_B = 0$

$0{,}427FL - H_A \cdot L - F/2 \cdot 2/3L + F \cdot L/2 = 0 \rightarrow \boxed{H_A = 0{,}594F}$

$\Sigma H = 0$

$-H_C + 0{,}594F + F/2 - F = 0 \rightarrow \boxed{H_C = 0{,}094F}$

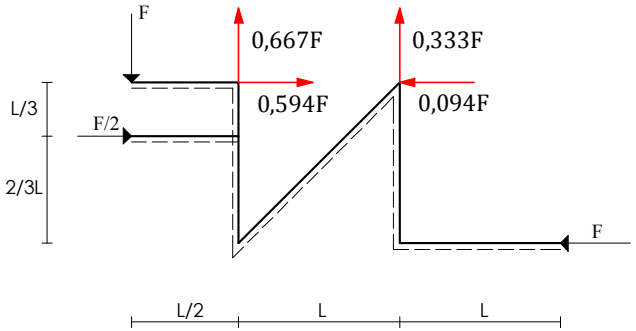
$\Sigma M_B = 0$

$-0{,}427FL + V_C \cdot L + 0{,}094F \cdot L = 0 \rightarrow \boxed{V_C = 0{,}333F}$

$\Sigma V = 0$

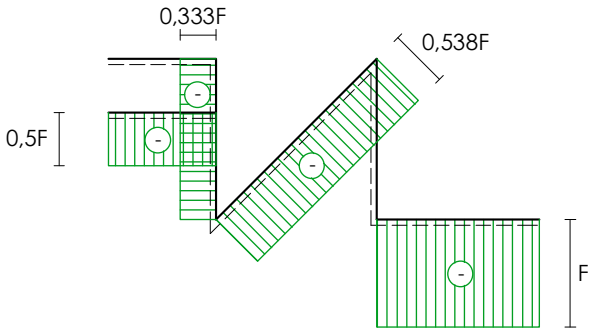
$V_A + 0{,}333F - F = 0 \rightarrow \boxed{V_A = 0{,}667F}$

Diagramma di corpo libero :



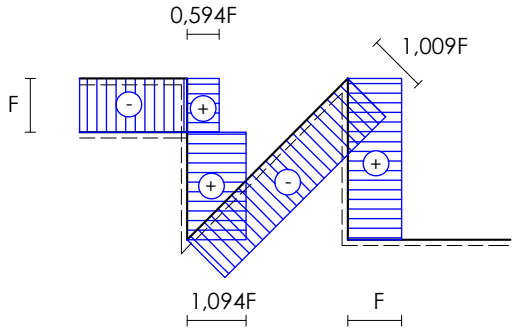
Diagrammi (N, T, M) :

$N \Rightarrow$

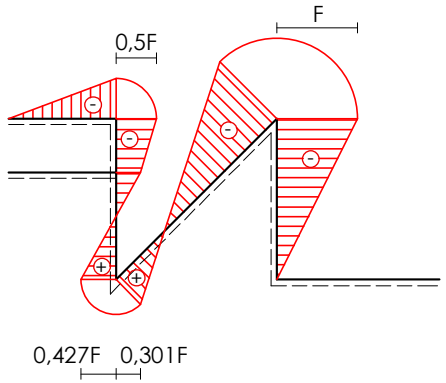


Esercizio N.37

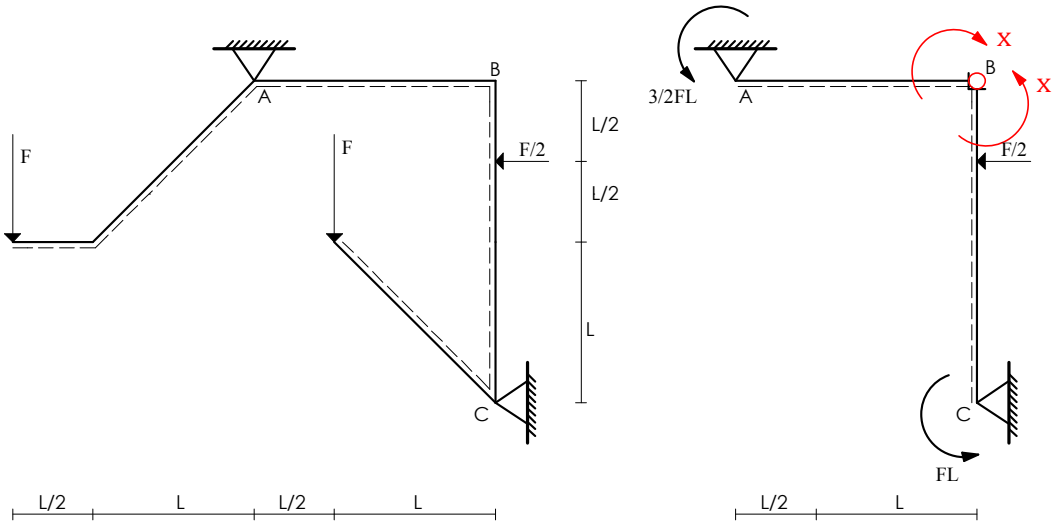
$T \Rightarrow$



$M \Rightarrow$

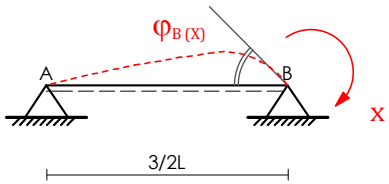


Esercizio N.38



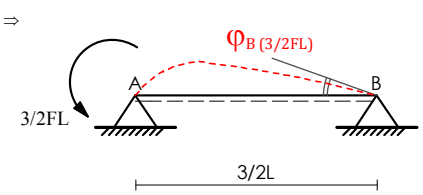
Eq. di Congruenza: $\boxed{\Delta\varphi_B = 0}$ ⇒ $\boxed{\varphi_{BA} = \varphi_{BC}}$ ⇒ $\varphi_{BA\,(X)} + \varphi_{BA\,(3/2FL)} = \varphi_{BC\,(X)} + \varphi_{BC\,(FL)} + \varphi_{BC\,(F/2)}$

$\boxed{\varphi_{BA\,(X)}}$ ⇒



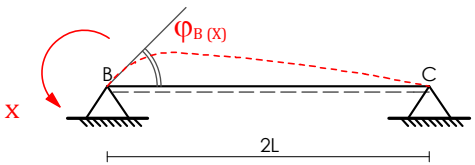
$$\boxed{\varphi_{BA\,(X)} = -\frac{X3/2L}{3\,EJ}}$$

$\boxed{\varphi_{BA\,(3/2FL)}}$ ⇒



$$\boxed{\varphi_{BA\,(3/2FL)} = -\frac{3/2FL(3/2L)}{6\,EJ}}$$

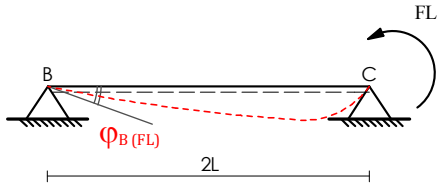
$\boxed{\varphi_{BC\,(X)}}$ ⇒



$$\boxed{\varphi_{BC\,(X)} = \frac{X2L}{3\,EJ}}$$

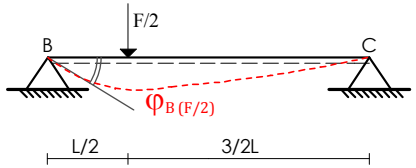
Esercizio N.38

$\boxed{\varphi_{BC\,(FL)}}$ ⇒



$$\boxed{\varphi_{BC\,(FL)} = -\frac{FL(2L)}{6\,EJ}}$$

$\boxed{\varphi_{BC\,(F/2)}}$ ⇒



$$\boxed{\varphi_{BC\,(F/2)} = -\frac{7(F/2)(2L)^2}{128\,EJ}}$$

Eq. di Congruenza:

$$\varphi_{BA\,(X)} + \varphi_{BA\,(3/2FL)} = \varphi_{BC\,(X)} + \varphi_{BC\,(FL)} + \varphi_{BC\,(F/2)} \Rightarrow$$

$$-\frac{XL}{2\,EJ} - \frac{3FL^2}{8\,EJ} = \frac{2XL}{3\,EJ} - \frac{FL^2}{3\,EJ} - \frac{7FL^2}{64\,EJ}$$

$$X = \frac{13FL^2}{192\,EJ} \cdot \frac{6\,EJ}{7L} = \boxed{0{,}058FL}$$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B

$\Sigma M_B = 0$
$0{,}058FL - H_C \cdot 2L - F/2 \cdot L/2 + F \cdot L = 0 \rightarrow \boxed{H_C = 0{,}404F}$

$\Sigma H = 0$
$H_A - 0{,}404F - F/2 - F = 0 \rightarrow \boxed{H_A = 0{,}904F}$

$\Sigma M_B = 0$
$-0{,}058FL - V_A \cdot 3/2L + 0{,}094F \cdot L + F \cdot 2L = 0 \rightarrow \boxed{V_A = 1{,}961F}$

$\Sigma V = 0$
$V_C + 1{,}961F - F - F = 0 \rightarrow \boxed{V_C = 0{,}039F}$

Esercizio N.38

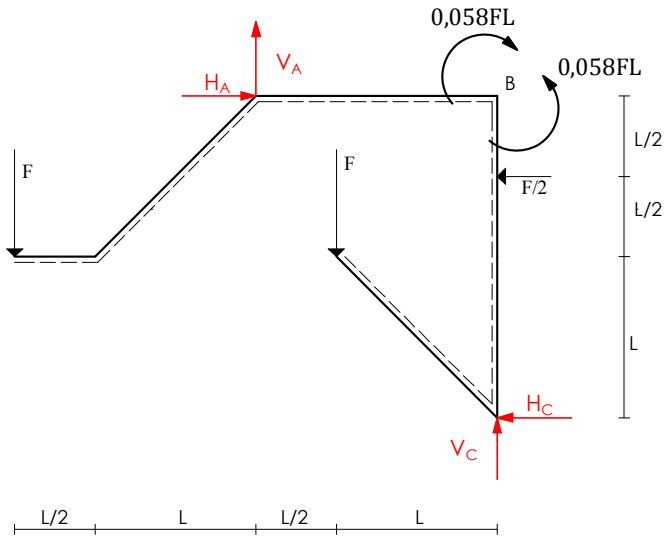
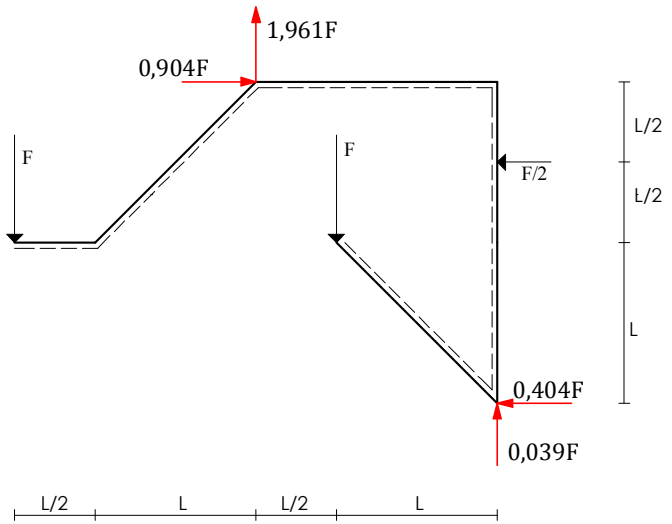


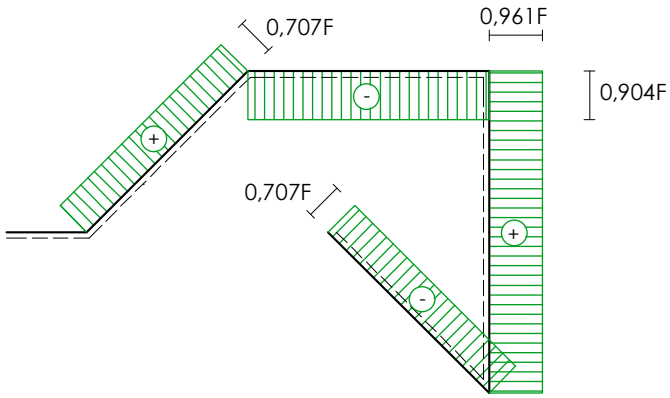
Diagramma di corpo libero :



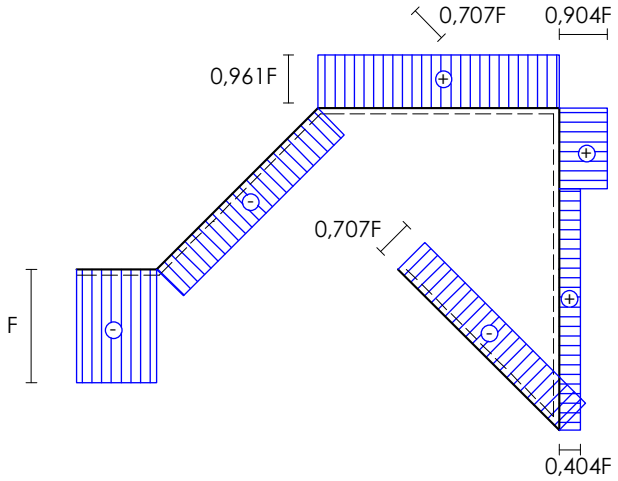
Esercizio N.38

Diagrammi (N, T, M) :

$N \Rightarrow$

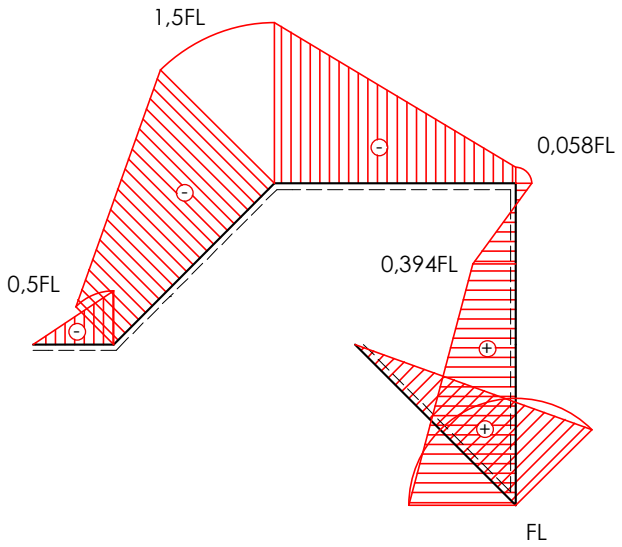


$T \Rightarrow$

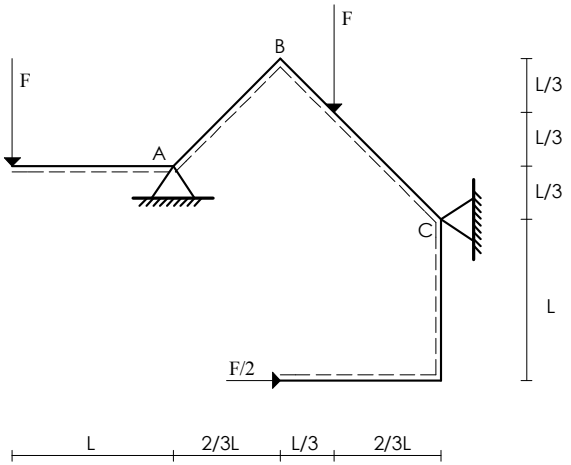


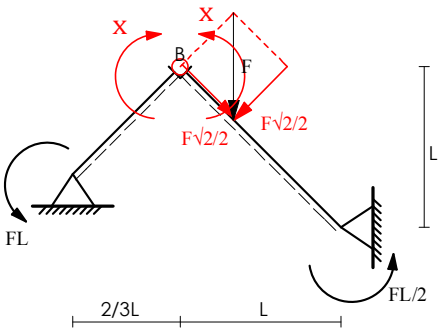
Esercizio N.38

$M \Rightarrow$



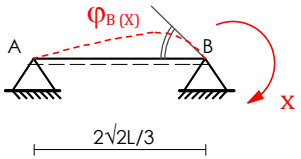
Esercizio N.39





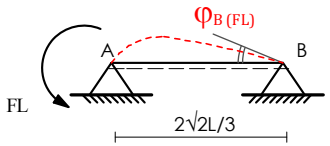
Eq. di Congruenza: $\boxed{\Delta\varphi_B = 0} \Rightarrow \boxed{\varphi_{BA} = \varphi_{BC}} \Rightarrow \varphi_{BA\,(X)} + \varphi_{BA\,(FL)} = \varphi_{BC\,(\sqrt{2}/2F)} + \varphi_{BC\,(X)} + \varphi_{BC\,(FL/2)}$

$\boxed{\varphi_{BA\,(X)}} \Rightarrow$



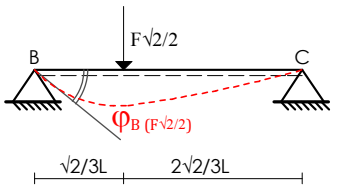
$$\varphi_{BA\,(X)} = -\frac{X2L\sqrt{2}/3}{3\,EJ}$$

$\boxed{\varphi_{BA\,(FL/2)}} \Rightarrow$



$$\varphi_{BA\,(FL)} = -\frac{FL2\sqrt{2}/3}{6\,EJ}$$

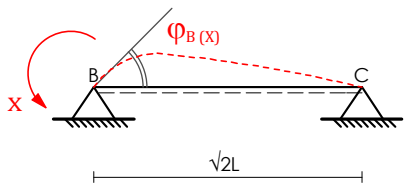
$\boxed{\varphi_{BC\,(F\sqrt{2}/2)}} \Rightarrow$



$$\varphi_{BC\,(F\sqrt{2}/2)} = -\frac{5(F\sqrt{2}/2)(\sqrt{2}L)^2}{81\,EJ}$$

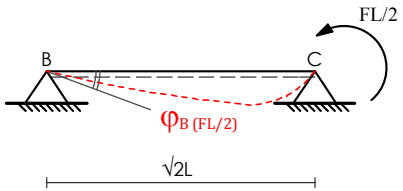
Esercizio N.39

$\varphi_{BC\,(X)} \Rightarrow$



$$\varphi_{BA\,(C)} = \frac{XL\sqrt{2}}{3EJ}$$

$\varphi_{BC\,(FL/2)} \Rightarrow$



$$\varphi_{BC\,(FL/2)} = -\frac{FL/2(\sqrt{2}L)}{6EJ}$$

Eq. di Congruenza:

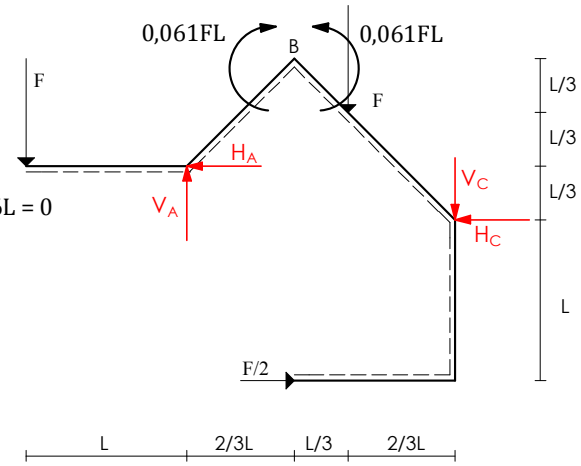
$$\varphi_{BA\,(X)} + \varphi_{BA\,(FL)} = \varphi_{BC\,(\sqrt{2}/2F)} + \varphi_{BC\,(X)} + \varphi_{BC\,(FL/2)} \Rightarrow$$

$$-\frac{2XL\sqrt{2}}{9\,EJ} - \frac{FL^2\sqrt{2}}{9\,EJ} = -\frac{5\sqrt{2}FL^2}{81\,EJ} + \frac{XL\sqrt{2}}{3\,EJ} - \frac{FL^2\sqrt{2}}{12EJ}$$

$X = 0{,}305FL \cdot 0{,}2 = \boxed{0{,}061FL}$

Calcolo reazioni vincolari incognite:

Eq. Ausiliaria : M_B



$\Sigma M_B = 0$
$0{,}061FL - F \cdot 0{,}333L - V_C \cdot L - H_C \cdot L + F \cdot L = 0$
$\rightarrow \boxed{H_C = -V_C + 0{,}728F}$

$\Sigma M_A = 0$
$F \cdot L - F \cdot L - V_C \cdot 1{,}666L - H_C \cdot 0{,}333L + F \cdot 0{,}666L = 0$
$\rightarrow \boxed{V_C = 0{,}318F}$

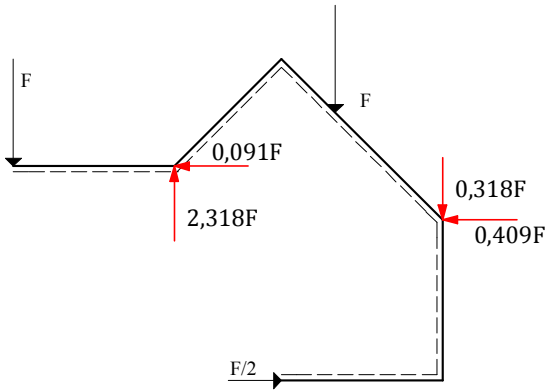
$\Sigma V = 0$
$V_A - F - F - V_C = 0$
$\rightarrow \boxed{V_A = 2{,}318F}$

$\Sigma M_B = 0$
$-0{,}061FL - H_A \cdot 0{,}666L + F \cdot 1{,}666L +$
$-2{,}318F \cdot 0{,}666L = 0 \rightarrow \boxed{H_A = 0{,}091F}$

$\Sigma H = 0$
$H_A + H_C - 0{,}5F = 0 \rightarrow \boxed{H_C = 0{,}409F}$

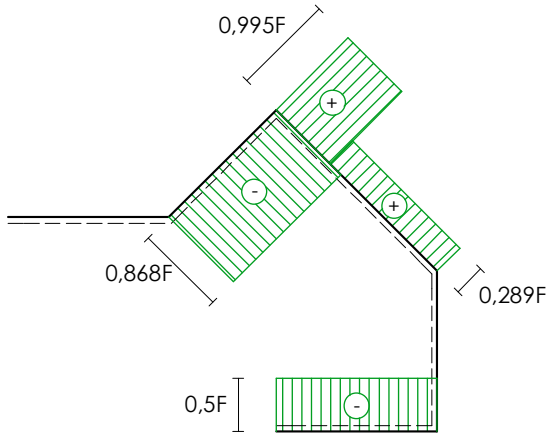
Esercizio N.39

Diagramma di corpo libero :

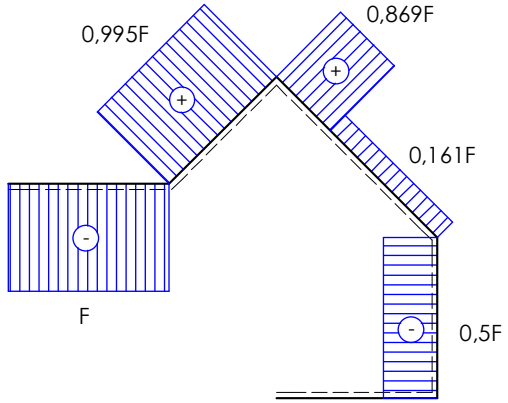


Diagrammi (N, T, M) :

$N \Rightarrow$



$T \Rightarrow$



$M \Rightarrow$